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THE

Columbian Arithmetician;

OR

NEW SYSTEM

OF

THEORETICAL AND PRACTICAL ARITHMETIC,

IN THREE PARTS:

PERFORMED IN

SIMPLE AND COMPOUND NUMBERS;

ADAPTED

TO THE COMMERCE OF THE UNITED STATES;

AND

Deſigned for the Inſtruction of Youth.

…………

BY AN AMERICAN.

…………

HAVERHILL, (MASS.)
PRINTED BY WILLIAM B. ALLEN,
And Sold at his Book-store, wholesale and retail, and by the principal
Booksellers in New-England.

1811.

Diſtrict of Maſſachuſetts, to wit.

District Clerk's Office.

Be it remembered, that on the eighteenth day of February, A D. 1811, *and in the thirty-fifth year of the Independence of the United States of America,* William Brown Allen, *of the ſaid diſtrict, has depoſited in this office the title of a Book, the Right whereof he claims as Proprietor, in the words following, to wit :*

THE COLUMBIAN ARITHMETICIAN ; OR NEW SYSTEM OF THEORETICAL AND PRACTICAL ARITHMETIC, IN THREE PARTS : PERFORMED IN SIMPLE AND COMPOUND NUMBERS ; ADAPTED TO THE COMMERCE OF THE UNITED STATES ; AND DESIGNED FOR THE INSTRUCTION OF YOUTH.

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Wm. S. SHAW,
Clerk of the District of Massachusetts.

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6 MAY 1959
LIBRARY

RECOMMENDATIONS.

THE subscribers, having perused the "Columbian Arithmetician," are of opinion that it contains many valuable improvements in the science of which it treats; particularly in Fractions, Proportion, Exchange, and the Extraction of Roots. We think it will be highly useful to the community in general, and particularly beneficial in schools.

ISAAC HACKER.
WILLIAM B. DODGE.
JOHN GRAY.
Salem, Mass. Aug. 20, 1810. ASA FLANDERS.

THE subscribers, having examined in manuscript, the Columbian Arithmetician, approve of the general plan; and think the author entitled to credit for much integuity, originality and improvement, in various parts of his work. We think the Book well calculated for the use of schools, and to promote the general knowledge of Arithmetic:—And we sincerely wish the author that liberal patronage, to which his talents, as a mathematician, justly entitle him.

ANDREW PEABODY, jun.
Beverly, Aug. 23, 1810. JOSIAH BATCHELDER.

HAVING inspected a new system of Arithmetic entitled the Columbian Arithmetician—I am fully satisfied, that it is better calculated than any I have seen, to subserve the instruction of youth, as well as the interest of the merchant and the man of business In the new method of introducing the knowledge of Fractions, and of Extracting Roots; and in the frequent exemplifications of the science of Geometry, are many ingenious and useful improvements. I sincerely wish that the industry and talents of the author may find ample reward, by meeting with general patronage and encouragement.

Beverly, August 23, 1810. ISAAC FLAGG.

Dear Sir,

I RECEIVED pleasure in perusing the Columbian Arithmetician, and must sincerely wish you success in the publication, from a conviction that it is calculated to subserve the interest of our youth. Among the numerous productions of the kind, I am induced to consider it as entitled to a respectable rank. The arrangement—the application of its rules to practical use, and the great correctness in performing the operations, recommend it to notice.

With esteem I am, &c. PETER EATON

Boxford, August 25, 1810.

HAVING perused in manuscript the "Columbian Arithmetician," we esteem it a work of much ingenuity and originality, well calculated for the improvement of youth in the science of arithmetic. Its consistent perspicuity, brevity and variety of matter, will no doubt render it useful. We would therefore cheerfully recommend it to public patronage.

JOSHUA JEWETT.
SAMUEL ADAMS.
SAMUEL JACKSON.

Rowley, Aug. 28, 1810.

Extract of a letter from Joseph Dana, Esq. Preceptor of the Newburyport Academy.

DEAR SIR,

I HAVE examined the COLUMBIAN ARITHMETICIAN, with as much attention as I have been able, and am well satisfied that it contains a greater variety of important matter than any work of the kind with which I have been acquainted: and have no doubt but that it will be found highly interesting and useful to instructors and to proficients in Arithmetic, and that it will reflect honor on the author, and our country.

I am Sir, respectfully yours, JOSEPH DANA.

Newburyport, Feb. 2, 1811.

THE Subscribers having examined in manuscript a new system of Arithmetic, entitled THE COLUMBIAN ARITHMETICIAN, are satisfied that it is better calculated than any other system extant to facilitate the progress of youth in the science of numbers. The ease, perspicuity and correctness in elucidating the fundamental rules—the illustrations in proportion, and the new method of extracting roots---The utility of the Tables of Exchange and the exemplifications of the science of Geometry, discover, not only great improvements upon former treatises, but contain a variety of ingenious and original matter; and we can recommend it to be peculiarly useful in Schools and Academies, and highly valuable to the community, and sincerely wish the author that approbation and encouragement so justly due to his talents and industry.

STEPHEN MINOT.
BAILEY BARTLETT.
LEONARD WHITE.
JOSHUA DODGE.
WILLIAM BATCHELDER.
JONATHAN C. HILL.
JOHN VARNUM.
JOSEPH HOVEY.
SAMUEL MERRILL.
JAMES C. MERRILL.

Haverhill, Feb. 15, 1811.

THE following gentlemen, among others, whose letters being lengthy we have not sufficient room to insert, have perused the Columbian Arithmetician, and given recommendations similar to the above.

Rev. ISAAC BRAMAN, A. M. Rowley.
Rev. JONATHAN ALLEN, A. M. Bradford.
Rev. ASAHEL HUNTINGTON, A. M. Topsfield.
Rev. MOSES DOW, A. M. Beverly.
Rev. JOHN SMITH, A. M. Salem. N. H.

PREFACE.

AS perfection in any art or science can never be attained, there is always room for improvement. And notwithstanding the valuable improvements, which have been recently made in the science of Arithmetic, the author of this treatise, after careful examination of the most celebrated publications of the kind, was thoroughly convinced that the progression might still advance with increasing light and utility; which is the only apology he has to offer for this humble attempt to promote the more general knowledge of so useful a science.

Some may suppose, taking into consideration the diverse characters employed and the numerous treatises written, that the subject is now exhausted, and inaccessible to further improvement; but numbers are the only perfect instruments we can handle, and the only means of ascertaining the true effect of any given cause, and until this can be done in the best possible manner, and no difficulty or obscurity attend any arithmetical operation, improvement awaits the science.

Resting assured that but few will doubt the possibility or necessity of improvement in every art and science, and that no amendment, however small it may appear, can, in a land which has hitherto been so highly favorable to growing science, be ungratefully viewed, shall with diffidence proceed to point out some particulars, to which it has been the general aim of the author to conform, and wherein it appears improvements have been, and may continue to be made; of which, if there be none found in this, it is confessed to be, not for want of pains taken, or condition of the science to receive.

There are two capacities which arithmetic has particularly to address, in point of utility; the first, is the youth, just beginning to count, whose abilities are insufficient fully to comprehend a rule, (from which in a short time he might receive singular advantages,) and whose progress can be promoted only by his instructor and by examples wrought out, or the different parts of the operation placed in a conspicuous manner. The second, is a person who has made some proficiency in the study, and would inquire into the various parts of the operation, apply it in practice, or review any part of his acquirement without an instructor; all which he will best perform by an apt rule, and still receive benefit, or at least no inconvenience from examples wrought out in the most simple manner. The former needs a variety of subjects, from which he may choose, and the latter, the same, to rouse his attention, and stimulate him to application; both of whom equally feel the many disadvantages of a great variety of examples proposed, to the exclusion of more necessary rules; since the higher branches of mathematics, are but an application of arithmetic, and each succeeding rule but practice on the foregoing. But if more examples than were thou sufficient, be deemed necessary, the scholar may very easily double number, having the answer to each given, by once inverting each tion, and a great variety by proportionally altering the given number and answer. A person incapable of doing this, or of proposing questions for himself, is equally incapable of receiving benefit from those proposed and not performed.

Perspicuity and conciseness, being the only qualities which render a work of this nature useful, have been the peculiar care of the author, and such a combination of both has been chosen as appeared most conducive to the instruction of the different capacities of youth: by giving explicit rules and difinitions, few and entertaining examples; as it is presumed that books designed for instruction, and to be useful, cannot be too plain if not too voluminous, nor too brief if not too obscure; hence formality and logical precision has been sacrificed to convenience and utility: the whole being so designed as to leave that necsssary room for exercise of the scholar's genius, something inviting before him for conquest; lest from the bare performance of numerous unmeaning and tedious operations, he might sink into absolute despondency, and acquire but a mere smattering of the science; since it is not the number of examples, with the performance of which he is fatigued, his alertness nor memory, that constitutes the mathematician, but a knowledge drawn from the due observance of rules, and consideration of their use and propriety, combined with that judgment which this science is calculated to improve, rather than enlarge the memory. For sake of brevity long operations have been omitted, and the given and required numbers, with the essential parts of the example, signs, &c. arranged in due conformity to the rule. Also, to the arithmetical signs there have been some additions made, to render them more plain and definite, and to save the often repetition of long words. The number of different positions in which figures may be advantageously used in their application to common business, is endless; and to explain more of their varieties, than will lead the student into a useful knowledge of the science, or to give a multitude of examples while he is acquainted with but few rules, is exacting more time and expense than the acquirement deserves, in comparison with others.

It is not expected that the contents of this treatise will be fully comprehended by the same slight application with which Temple's Primer, or a trifling superficial knowledge of the simple rules, may be learned; for it is designed to contain nearly all the arithmetical knowledge necessary in the common affairs of life. Though perhaps the same subjects here contained, may be learned in as short a time and with as much ease, as those contained in any other treatise of the kind. Yet obscurity on some particulars in a work of this nature is unavoidable, and is, at the present state of improvement, inseparable from some of the subjects treated of; also error in the first impression of a mathematical work is equally unavoidable; and perhaps it may contain improprieties, notwithstanding the careful attention which has been paid to remedy all the evils to which, from the nature of the science, it is subject; communications of which, or any improvement which may be suggested will be thankfully received.

With regard to the arrangement of rules, each instructor or pupil will follow his own taste; as it is of little importance what subject ws the acquirement of the four simple rules and rule of three: but a systematic arrangement, such as would bring every subject into its proper channel, was designed; and the whole system of mathematics being but an application of the four simple rules of arithmetic, several subjects are introduced under those rules and the rule of three, which are by others treated of in a different manner; and such subjects as require an absolute combination of rules are introduced after those performed by a single rule, and each preceding or succeeding another, as it is supposed to be more or less useful. And with a view to facilitate the acquirement of arithmetic, and to bring more fully into use the most important subject it contains, (without violation of rule or system,) both integers and decimals, because they differ only in value, increase and decrease in the same ratio, &c. have been considered as belonging to the same rule. The contractions and application of rules were not designed for the strict attention of young scholars; but as they may be of utility to those who have acquired an adequate knowledge of the principle rules, it was thought expedient to insert them. Under numeration the tables of weight, measure, &c. with a variety of useful proportions, are introduced that the scholar might more easily become acquainted with them as they often occur throughout the work: under this head there are also several tables for finding the contents of divers kinds of figures, the specific gravity of substances, &c. which would no where else so conveniently occur, with regard to utility. And under application of the simple rules is given a rule for easily extracting the roots of all powers, in lieu of that complicated extraction which has hitherto exacted so much time and labor. It is not expected that this rule will be very easily learned by a young scholar without an instructor. Proportion through the following treatise is supposed to be simple and direct, and a sufficient number of questions stated therein, which are usually termed inverse and compound, perhaps to render it satisfactory to the inquirer, that there can be no convenience experienced, but many disadvantages, from dividing proportion into so many different kinds as are by some thought necessary, and which only serve as so many needless embarrassments to the learner. Proportion is of such a nature that it cannot literally admit of a compound, (for, though the terms may be composed of several parts, it is an improportion until it be rendered simple;) neither will it admit of being inverse, between a series of numbers, compared with another between which the proportion is direct; for, all the parts of any cause making the whole, must bear such proportion to its whole effect, as all the parts of another similar cause does to its whole effect, (or all its parts, conjunctly,) in every case. Duodecimals and all numbers whose division is not decimal, are in the following supposed to belong to the four compound rules, and under division a new method of dividing duodecimals, &c. is given. The knowledge of fractions, (so useful while our present system of weights, measures, &c. continue,) and that of the indispensable science of geometry being previously initiated in every convenient place, it was not deemed necessary to continue them to a great length as separate subjects; the design, of dispersing those subjects as above mentioned, is that persons perusing the following treatise may not easily as usual neglect those very important branches o mathematical knowledge, and to give to the rules that useful applic tion so necessary to gain the attention of the learner. Position, al tion, progression, evolution, &c. are subjects of which a complete investigation, such as would sufficiently elucidate all their intricacies, to the perception of an ordinary capacity, would require the work of volumes; but from what is given, a sufficient knowledge, as it respects utility, may be acquired. Exchange, with different parts of the world, is very essential to the merchant and traveller; but under this head nothing more seems necessary, than the simple value of currencies, weights, &c. or the proportions between these and those to which they are to be reduced and a few examples; but since no two, of the best authors can be found to agree on this subject, it is very difficult to choose, and from the present instability of foreign monies, it is impossible to determine the true par of exchange, with all countries; therefore a rule is given for ascertaining the value of all real coins, or of gold and silver, from their specific gravity. The touch on subjects foreign to arithmetic, may, in many instances, be found useful, in common business, as well as to inspire the industrious youth with a relish for further researches into the higher branches of mathematics. Where a rule in this treatise, on any material point, essentially deviates from that used by others, the common method is also inserted; that the scholar may be able to choose from the immediate contrast; as it is designed not to obscure any light, heretofore thrown on the subject, nor leave any thing untouched, which has been previously handled, necessary to be noticed in arithmetic. Yet it must experience many inconveniences and local prejudices, on account of its wide, but supposed necessary, deviation from others; as all ranks of society have their aversions, prejudices and peculiarities; and as it is impossible to unbias the minds of those who have been accustomed to repose implicit confidence in the judgment of those from whom they received their elementary tenets.

Arithmetic in its present state appears to be naturally divided into three parts. The first of which is numeration or the art or act of ascertaining and expressing any value or quantity in any form. The second, the art of performing operations with simple numbers. The third is arithmetic in compound numbers; which in process of time will be no more than nominal: for numbers must inevitably undergo great and valuable change; as the revolution, begun long before the introduction of the Arabic characters, which greatly advanced it, is not half completed; as may be readily seen from a review of the progress of arithmetic, and the present irregularity of our best numeral systems, on account of their many vulgar divisions of value and quantity, which are all we have to compute, with our Arabic decimals; and if the quantity partake of less than unity we must have recourse to parts irregular as the daily productions of chance, or be unable to ascertain the resulting value in known denominations. A perusal of the tables of weight, measure, &c. which we daily use, and a knowledge of decimals is all that is necessary to convince every person of the inconvenience and impropriety of such divisions and of their more absurd different values bearing the same name, (as gallon, tun, second, &c.) The only divisions of any propriety now in use are such as increase in the same ratio; and those (if not decimal) falter and stumble at the integral point, where their disagreeable ratio is lost in decimal. The general reason why we ascertain the quantity is that its price may be determined; and until the dimensions of all magnitudes or quantities be determined by a measure of length, whose division corresponds with that of the coin, in which the value of the quantities produced by a regular involution of the length, is given, and all ratios, between which there is any connexion, be correspondent, we shall be perpetually embarrassed with innumerable tedious reductions and intricate proportions. For this ratio, none can be compared with the decimal, to which are adapted, the characters by which we compute, all numbers from unity upward, and the American coin, which is (with others similarly divided) the best in the world; and which has, in our best American productions, been very much neglected, while the devoted labor of thousands has been bestowed on that, which is, only by pride and prepossession withheld from oblivion: and which will, with all others, and the hundreds of different and ill divided weights and measures, be lost in the aggrandizement of one complete standard of decimal coin, weight and measure; which will lessen the labor of acquirement in this science more than any other improvement it has ever received: the method of placing the decimal point being the only knowledge necessary, separate from that of performing operations in whole numbers.

A system as above mentioned of decimal division of quantity, which was calculated to explain the various operations of decimals, to show the propriety of corresponding divisions, &c. a sketch of arithmetical trigonometry, a demonstration of rules, and a few other subjects, have been omitted, as the size of the volume already exceeds what was designed, and as the task of publishing has been unexpectedly lengthy and tedious, from the difficulty of correcting a work of this kind, and as the copy was better calculated for an octavo page, and therefore comprises more than usual in the same space; which may by some be thought an error, if so, it is hoped it will be a profitable one, and that the size, price or contents of the volume will not be dissatisfactory to its purchasers.

From a conviction of the inadequate knowledge contained in our present systems of arithmetic, and of the many disadvantages experienced therefrom, the author was prompted to the undertaking of the arduous task, which has been executed with a fidelity suited to its importance. And how far it may be deserving of that patronage and encouragement to which it is so liberally recommended by those gentlemen who have honored it with their names, for which the highest expressions of gratitude are tendered, and what part, it may contain, of that merit, which with unremitted diligence has been sought, is left to the candid decision of a liberal and impartial public.

Haverhill, Feb. 1811.

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Use and Explanation of Characters.

THE characters or ſigns uſed, for ſake of brevity, in the following treatiſe, and thoſe uſed by others, ſignify the ſame that the phraſes or words do which are here annexed to them.

0 cipher; 1 one; 2 two; 3 three; 4 four; 5 five; 6 ſix;
7 ſeven; 8 eight; 9 nine;

Signs;	*ſignifications,*	*explanations.*
$=$	*is equal to, or make;*	as, $1\ \$ = 100$ cents, i.e. $1\ \$$ is equal to 100 cents.
$+$	*added to, or add;*	as, $5+5=10$, i.e. 5 added to 5 is equal to 10.
— or —	*ſubtracted from, or take;*	as, $3 - 5 = 2$, i.e. 3 ſubtracted from $5 = 2$. from $5 - 3 = 2$.
$\times$ or x	*multiplied by, or times;*	as, 3 x 2, or 2 x 3 = 6, ie. 3 times 2 or 3 multiplied by 2 = 6.
$\div$	*divided by;*	as, $6 \div 2 = 3$, i.e. 6 divided by 2 $= 3$. $2 \div 4 = ,5$ or ${}_{4}$.
$\frac{6}{2}$ or 2)6(	*diviſion;*	as, $\frac{6}{2} = 6 : 2$. 2)6(3. $\frac{2}{4} = 2 \div 4 = ,5$.
: :: : or : :: :	*geometrical proportion;*	as, 2 : 4 :: 3 : 6, i.e. as 2 is to 4 ſo is 3 to 6.
.. :: ..	*arithmetical proportion;*	as, 2..4::3..5, ie. 2 is to 4 as 3 to 5 arithmetically.
4^2 or $\overline{4}\vert^{2}$	*ſquare (of 4) required;*	as, $4^2 = (4 \times 4 =) 16$. $\overline{2}\vert^{2} = 4$. $8^2 = 8 \times 8$.
2^3 or $\overline{2}\vert^{3}$	*cube (of 2) required;*	as, $2^3 = (2 \times 2 \times 2 =) 8$. $\overline{4}\vert^{3} = 64$. $3^3 = 3 \times 3 \times 3$ or 27.
2^m or $\overline{2}\vert^{m}$	*any power (of 2) required;*	as, $2 = 2 \times 2$, $2 = 2 \times 2 \times 2$, or $2^4 = 2 \times 2 \times 2 \times 2$, &c.
${}^{2}4$, $\sqrt[2]{4}$, $\sqrt{4}$ or $\overline{4}\vert^{\frac{1}{2}}$	*ſquare root (of 4) required;*	as, ${}^{2}4 = 2$. $\sqrt{9} = 3$. $\overline{16}\vert^{\frac{1}{2}} = 4$. ${}^{2}25 = 5$.
${}^{3}8$, $\sqrt[3]{8}$ or $\overline{8}\vert^{\frac{1}{3}}$	*cube root (of 8) required;*	as, $8 = 2$. $\sqrt[3]{8} = 2$. $\overline{27}\vert^{\frac{1}{3}} = 3$.
${}^{m}8$, $\sqrt[m]{8}$ or $\overline{8}\vert^{\frac{n}{m}}$	*any root (of 8) required;*	as, ${}^{2}8$, or ${}^{3}8$, or ${}^{4}8$, &c.

The number with the ſign or figure prefixed is accounted the expreſſion of the root or power required, and is equivalent thereto.

The character prefixed to the number is called the *radical ſign* and the figure the *index*.

A *root* is the firſt term and ratio of a *geometrical* progreſſion, its products are powers or terms.

$\overline{5-2}$ *vinculum*, or *connexion* of the numbers over which it is drawn; as $\overline{5-2}+1=4$, i.e. the difference between 5 and 2 added to $1=4$. $5-\overline{1+2}=2$, i.e. the ſum of 2 and 1 ſubtracted from $5=2$. $\overline{8-2}\times 4=16$, $8\div\overline{2\times 4}=1$.

$=y$	equality.	— or ——	ditto.	dn	denomination.
A	anſwer, or reſult.	$+d$	added.	$+g$	adding.
$+n$	addition.	$-t$	ſubtract.	$-d$	ſubtracted.
$-g$	ſubtracting.	$-n$	ſubtraction.	$-nd$	ſubtrahend.
$-r$	remainder.	$-ng$	remaining.	$\times y$	multiply.
$\times d$	multiplied.	$\times g$	multiplying.	$\times n$	multiplication.
$\times r$	multiplier.	$\times nd$	multiplicand.	pt	product.
$\div e$	divide.	$\div d$	divided.	$\div g$	dividing.
$\div n$	diviſion.	$\div r$	diviſor.	$\div nd$	dividend.
qt	quotient.				

When the above ſigns are to repreſent the plural number, *s* is commonly added; as, *As* for anſwers, x*rs*, &c.

Geometrical Definitions.

Contents, is the quantity of matter, ſubſtance, weight, or what is compriſed within ſides, lines or points.

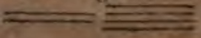
Sides are the edges or lines of a ſurface, or faces of a ſolid.

Mean area, length, breadth, &c. is the area, length, &c. reduced to a medium, or ſuch as the figure will bear from end to end or from ſide to ſide, without enlarging or depreciating its ſize.

A *point* is a ſmall mark or end, without dimenſions; as ·

Dimenſions are the limits of the contents, & called lines;

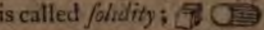
A *line* is the diſtance between two points, and has length only; a *right line* is the ſhorteſt diſtance between two points, as, ——. ——.

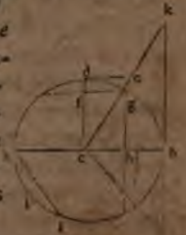
Parallel lines are right lines equally diſtant from each other in all places; 

A *curved or arched line* is a line bending between two points; ◡ ⌒ ſo if continued will encloſe a circular plane; ○

A *plane or ſurface* of any kind has two dimenſions, length which is its greateſt mean extenſion, and breadth its leaſt; its contents is called *area*; ○ ▭ a *plain* is a flat or level figure; and a ſurface is the outſide of any thing.

A *figure* is the ſhape or image of any thing, and is conſidered either as a ſurface or ſolid.

A *ſolid or body* has three dimenſions, length, breadth and thickneſs, its contents is called *ſolidity*; 



A *circle* is a round figure, whoſe *circumference* or *periphery*, which is the curve line or boundary, is conſidered as being divided into 360 degrees or equal parts, each of theſe into 60 minutes, &c. 60 degrees is the diſtance from the *centre* or middle to the periphery, and is called the *radius* or ſemidiameter; as *a c*, double this is the *diameter*; as, *a b*.

An *arch* or *arc* is any part of the circumference of a circle; as, *a l i*, or *a d*.

A *chord* is a right line drawn from one end of an arch to the other; as, *a i*.

A *ſine* is half the chord of an arch; as, *g h* is the ſine of the arch *g b*.

A *verſed ſine* is that part of the radius which is contained between the ſine and circumference; as, *bh*.

A *tangent* is a right line raiſed perpendicularly on one end of the diameter, and is terminated by a line drawn from the centre through the other end of the arch, which is called,

A *ſecant*; as *bk* is the tangent and *ck* the ſecant of the arch *bg*.

The *complement* of an arch is what it wants of 90 degrees or $\frac{1}{4}$ of a circle; as, *dg* is the complement of the arch *bg*.

The *ſupplement* of an arch is what the arch wants of 180 degrees or $\frac{1}{2}$ of a circle; as *adg* is the ſupplement of the arch *gb*.

The ſine, tangent or ſecant of the complement of an arch is called the *co-ſine co-tangent* or *co-ſecant* of the arch; as, *fg* is the co-ſine, *d e* the *co-tangent* and *ce* the *co-ſecant* of the arch *gb* or they are the ſine, tangent and ſecant of the arch dg.

The ſine, tangent or ſecant of an arch is alſo the ſine, tangen *or ſecant of an angle* containing the ſame number of degre

An *angle* is a corner, formed by the meeting of two lines; and if one of thoſe lines fall directly on the other they form a *right angle* equal to 90 degrees or $\frac{1}{4}$ of a circle, on each ſide of the line ſo falling this line is called a *perpendicular*; ⊥ an angle leſs than 90 degrees is called an *acute* angle; ∠ a larger is *obtuſe*; ∠ and both are *oblique*.

A *triangle* is a figure having three ſides and angles, the whole contains 180 degrees; △ △

A *right triangle* has one right angle and two acute angles, the longeſt ſide is called the hypothenuſe, the next or bottom line the baſe, and the other the perpendicular; ◺ ◿

An *oblique triangle* has three oblique angles, the longeſt or bottom line is commonly called the baſe and the ſhorteſt line that will reach from the oppoſit angle to the baſe is called the perpendicular; △ *acute triangle*; △ *obtuſe triangle*.

A *ſemi-circle* is half of a whole circle; ◓ ◒

A *ſector* is a part of a circle included between an arch line and two radii; △ ◔

A *quadrant* is $\frac{1}{4}$ of a circle; ◔ = the ſine of 90°

A *ſegment* is a part of a circle included between an arch and a *chord line* or line cutting the circle into 2 unequal parts; ⌓ ⌓

A *regular polygon* is a figure having equal ſides and angles; ⬡ ⬠

A *ſquare* has four equal ſides and right angles, and contains 360 degrees equal to a circle; alſo a number multiplied into or by itſelf produces the ſquare of that number, being the contents of the ſquare; □

A *cube* has ſix ſquare ſides, and is a ſolid regular body; alſo a number multiplied by its ſquare produces the cube of that number, being the cube's contents; ❒

Extract from the Permanent Law of Numbers.

It is Decreed and Ratified by the nature of Figures,

That their *relative value* ſhall be determined by the *place* they occupy, and their *poſitive value* by their *ſhape* and ſubſtance or *capacity repreſented*; viz. that a figure, in the firſt place called *units*, ſhall have its poſitive ſimple value, according to its *ſhape*, &c. and in the ſecond place called *tens* in a ten fold ratio, ten times as much as if it were in the firſt; but if in the place called *tenths*, one tenth part as much as in the firſt, &c. (ſo 55,5 \$. (d. c. lb. yd.) the relative value being a number of tens, units and tenths, the ſhape of each 5 and ſubſtance repreſented \$; the poſitive value therefore, is five tens, five units and five tenths \$, or fifty five \$ and five tenths.)

That the *ſum*, of numbers *added*, ſhall be equal in value to all *its* parts, and occupy the places of the numbers of which *it* is formed together with their natural increaſe; (as, 81 + 90 = 171; here the ſum has the places of 81 & 90 viz. units & tens & the increaſe 1, being one place, & is equal to 81&90.)

That the *remainder* or *difference*, of numbers *ſubtracted*, ſhall be ſufficient to increaſe the leſs number to the greater, and occupy the places of the number ſubtracted, or of that from which *it* is taken, excepting the decreaſe; (as, 14 — 8 = 6; here 6 has the place of 8, units belonging to both, it alſo has the place of 4 but not of 1, ſo the decreaſe is one place, 8 + 6 = 14.)

That the *product*, of numbers *multiplied*, be as many times larger than one of thoſe numbers or factors as the other contains units, or as many times ſmaller than one number as the other is ſmaller than an unit, and ſhall occupy a place repreſented by the names, of the factors employed, placed one before the other, with the ſign × or *of* between them, and by the ſum or difference of the arithmetical indices of the factors;* i. e. the names, of the places multiplied with the ſign × or *of* between them, repreſent the firſt figure or place of the product, and their increaſe the other; (as, $\overset{1}{20}\overset{+}{\times}\overset{2}{700}\overset{=}{=}\overset{3}{14000}$ the firſt figure of the product 4 is repreſented by tens of hundreds, and by the ſum of the indices 3, and the ſecond place 1 by the increaſe which is one place, and is never more; ſo tens of tens of hundreds repreſent the higheſt place in the product=tens of thouſands. $\overset{1}{30},\text{x}\overset{-1}{,3}=\overset{=0}{9,0}$ here 9 is repreſented by tens of tenths=units. $\overset{1}{40}\text{ x }\overset{-\;2=1}{,03}=$ to a product whoſe firſt figure is repreſented by tens of hundredths=tenths, and ſecond or laſt by the increaſe one place. $\overset{1}{,7}\text{ x }\overset{2=3}{,45}=$ to a product whoſe firſt figure is repreſented by tenths of hundredths=,001, and ſecond by tenths of tenths, viz. 7 tenths of 4 tenths=,08 to this product belongs the increaſe of 5 and 4=,23 ſo the whole is ,315.)

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* Arithmetical indices are here conſidered as the repreſentatives of the ſeveral degrees in integers and decimals; as $\overset{3\,2\,1\,0\,1\,2\,3\,4\,5\,6\,7\ \text{Indices}}{3400,12340,4\cdot}$ The ſum of the indices of the factors is the index of the firſt figure of the product, when the numbers x'd are both integers or decimals; as, $\overset{3}{2000}\overset{+}{\times}\overset{2}{700}\overset{=}{=}\overset{5}{1400000}$ or $\overset{3}{,002}\overset{+}{\times}\overset{2}{,07}\overset{=}{=}\overset{5}{,00014}$; but their difference is the index of the firſt figure of the product, when one factor is an integer and the other a decimal; as $\overset{3}{2000}\overset{-}{\times}\overset{2}{.07}\overset{=}{=}\overset{1}{140}$ or $\overset{2}{700}\overset{-}{\times}\overset{3}{.003}\overset{=}{=}\overset{1}{2,1}$ — the product always belongs to that ſide of the point which has the larger index.

B 2

That the *quotient*, of numbers *divided*, be as many times larger than an unit as the dividend is larger than the diviſor, or as many times ſmaller than an unit as the dividend is ſmaller than the diviſor, and ſhall occupy a *place* as much higher than unity, as the dividend is higher than the diviſor, alſo as much lower than unity or 1, as the dividend is lower than the diviſor; (which is repreſented by the difference or ſum of the arithmetical indices†) i. e. the firſt quotient figure muſt occupy a place as far diſtant from units, (to the right or left,) as the higheſt or left figure in the dividend is diſtant from that of the diviſor, (right or left,) and no farther, the decreaſe excepted, which is never more than one place to the right, and that only when the left of the diviſor exceeds that of the dividend; (ſo the names of the ÷*r* and *qt.* repreſent the ÷*nd*; as, 2)60(30 here the *qt.* 30 is as much higher than units, as the ÷*nd* 60 is higher than the ÷*r* 2 i. e. one place. 200)2,00(,01 here the *qt.* 1 hundredth is as much lower than units as 2 is lower than 200. 30,),00024(,000008 here 8 is as far from units as 24 is from tens, or 30,=4 places and 1 decreaſe.

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† Division being the reverse of multiplication, and addition the reverse of subtraction, the difference between the indices of the divisor & dividend is the index of the quotient, when the highest figures in the divisor and dividend are both integers or decimals; as, $\overset{2\,-\,1=\;1}{200,)4000,(20.}$ or $\overset{2\,-\,3=1}{,02),004(\,,2}$; but their sum is the index of the first figure of the quotient when the highest figure of the divisor or dividend is an integer and the other a decimal; as, $\overset{3+2=5}{,003)400.(200000,}$ or $\overset{5\;+\;1\;=\;6}{200000),4(,000002}$—the first figure of the quotient *is always to the right* of the point when the dividend is larger than *the divisor,* and always to the left when the dividend is less than the *divisor,*

ARITHMETIC.

ARITHMETIC is the ſcience of calculating by numbers; it explains the nature of figures and demonſtrates the principles and cauſes on which their rules depend.

Arithmetic is alſo the art of expreſſing numbers by certain characters and producing different effects from the ſame characters or the ſame effects from different characters; for which purpoſes it is eſſentially divided into three PARTS, viz. *Numeration*, *Simple* and *Compound Arithmetic*; each of the two latter *diviſions* is ſubdivided into four principal RULES; viz. *Addition*, *Subtraction*, *Multiplication* and *Diviſion*; and the values of their reſults, &c. are ſeverally aſcertained by *Numeration*.

PART 1st. *NUMERATION.*

Numeration is the art or act of aſcertaining and expreſsing the value of figures or numbers, according to their definite meaning, and local ſituation or diſtance from the ſtandard *unity* in decimal or ſimple arithmetic, and according to the diviſion of the integer or unit in vulgar or compound arithmetic.

Decimal or Simple Numeration is that in which the relative value of every *figure* is determined by its diſtance from units; which is a term uſed to ſignify the fixed place of the firſt figure at the right hand of integers or at the left of the decimal point, in which a figure has merely its ſimple value; the ſecond place is called tens in which a figure has ten times its ſimple value; the third hundreds, &c. The firſt place at the right of the point is termed tenths in which a figure has one tenth of its ſimple value, i.e. tenths of the integer; the ſecond place is called hundredths, &c. So each figure increaſes or decreaſes by removal toward the left or right in a ten-fold proportion, and the whole ſum may be increaſed or decreaſed ten-fold by *removing the point* one place to the left or right. The above *remarks appear* in the following tables.

TABLE 1. NUMERATION OF INTEGERS AND DECIMALS.

m m m m m llions of i lions of millions.

Septillions		or m
sextillions		or m
Quintillions		or m
Quatrillions		or m
trillions	2	or m
hunds of thous of bs	7	
tens of thous of bilns	6	
thousands of billions	8	
hundreds of billions	9	
tens of billions	1	
billions	0	or
hunds of thous of ms	3	
tens of thous of ms	5	
thousands of ms	6	
hundreds of ms	2	
tens of ms	0	
millions	9	
hunds of thousands	4	
tens of thousands	7	
thousands	6	hund of E
hundreds	0	E of E
tens	8	E
units	4,	$
tenths	5	d
hundredths	5	c
thousandths	0	m
ten thousandths	7	
hundred thousandths	9	
millionths	6	
billionths		or m
trillionths, &c.		or m

th of th of th of th of th of th th. of th. milliouths

2. — COMPARATIVE VALUE OF INTEGERS AND DECIMALS.

10 hundredths=	1 tenth, hth.	th.	=10hth.	=1th.	=,1u.	=,01t	=,001
10 tenths	1 unit	u	100	10	1	,1	,01
10 units	1 ten	t	1000	100	10	1	,1
10 tens	1 hundred	hd	10000	1000	100	10	1,
10 hundreds	1 thousand	thd	100000	10000	1000	100	10,

3. NUMERATION OF FEDERAL MONEY.

	Signs.	m.	c.	d.	$	E	Grains Troy.
10 mills=1 cent	m.c.	=10	= 1	= 1	=,01	=,001	=208, copper
10 cents 1 dime	d.	100	10	1	,1	,01	37,364 fine silver
10 dimes 1 dollar	D. or $	1000	100	10	1	,1	409,78 stand silv.
10 dollars 1 eagle	E.	10000	1000	100	10	1	246,268 fine gold 22,388 alloy.

Vulgar or Compound Numeration is the numbering of fractions or irregular parts; in which the relative value of every *part* is determined by the number of parts the integer or unit is divided and ſubdivided into; which parts are called the diviſor or denominator; which in Engliſh and other diviſions is known by divers names; as, twentieths are known by ſhillings, 12ths of ſhillings (or 20ths) by pence, 16ths and 12ths by ounces and inches, &c. as

$\frac{10}{20}$ $\frac{\text{ten}}{\text{twentieths}}$ of a pound $=\frac{10}{\text{shillings}}$, $\frac{8}{12}$ of a ft $=\frac{8}{\text{inches}}$ or 8 in 16 lb $=\frac{4}{\text{oz}}$.

In ſuch caſes as the preceding it is only neceſſary that the dividend or numerator, which is the value of the given ſum, (in thoſe parts of the integer,) ſhould be expreſſed in figures; *but a diviſor that is not* generally known as above ſhould be *placed under a line* with the dividend or given value over it; as $\frac{4}{5}$ $\left(\frac{\text{nu}}{\div r}\right)=\frac{\text{four}}{\text{fifths}}$ or four parts of five. $\frac{34}{40}$=34 parts of 40.

Vulgar Fractions are of three kinds; viz. *Regular or proper fractions*, i.e. the ÷r the largeſt; as, $\frac{4}{5}$, $\frac{9}{12}$, $\frac{110}{120}$. *Irregular or improper* fractions, i.e. the ÷r not the largeſt; as, $\frac{1}{21}$. *Compound* fractions, i.e. fractions of fractions or parts of fractions; as, $\frac{3}{4}$ of $\frac{1}{1}$ of $\frac{1}{20}$ of a pound = 3 farthings. $\frac{10}{12}$ of $\frac{1}{12}$ feet = 10 ſq. in. or $\frac{10}{144}$ ft. Compound fractions include all the following,

Tables of Engliſh Money, Weights and Meaſures,

in which the *proportions* are given which one bears to another, and the *value* of each one compared with another.

TABLE 1

STERLING MONEY

			signs	Proportions, values or multipliers. reduced to Federal Money. D	cts.	D
4 farthings	=	1 penny	f. d.	$=\frac{1}{54}$	=1,6518 =	,018513
12 pence		1 shilling	s	$\frac{2}{9}$	22 2222	,222222
20 shillings		1 pound	£	$\frac{40}{9}$	444,4444	4,414414

2. TROY WEIGHT

			--- to lb Avor	val of stan silv in dls.	val stan. gold dls
,0037 in water	1 grain	gr	$\frac{24}{168000}$	,0024	,0372
24 grains	1 penny weight	pwt	$\frac{576}{168000}$	,0585	,893335
20 pwts	1 ounce	oz	$\frac{576}{8400}$	1.1713	17.866713
12 ounces	1 pound	lb	$\frac{576}{700}$	14.0560	214 40056

3. APOTHECARIES WEIGHT

			--- to lb Troy	gr Troy	inches of pure water.
20 grains	1 scruple	gr ℈	$\frac{1}{288}$	20	,07904
3 scruples	1 dram	ʒ	$\frac{1}{96}$	60	,23712
8 drams	1 ounce	℥	$\frac{1}{12}$	480	1,89696
12 ounces	1 pound	lb	$\frac{1}{1}$	5760	22,76352

4. AVORDUPOIS WEIGHT

			--- to lb Troy	inches of pure water.	glns proof spirits
16 drams	1 ounce	dr oz	$\frac{700}{9216}$	1 729	,008087
16 ounces	1 pound	lb	$\frac{700}{576}$	27 664	,129[illegible]
28 pounds	1 quarter	qr	$\frac{1225}{36}$	774 61	3,62[illegible]
4 qrs or 112 lbs	1 hd weight	Cwt	$\frac{1225}{9}$	3098 44	14,[illegible]
20 cwts	1 tun	T	$\frac{24500}{9}$	61968,8	289,84

5. CLOTH MEASURE			reduced to feet	inches	seconds
2¼ inches =	1 nail	in na	= $\frac{3}{16}$ =	2,25	27
4 nails	1 quarter	qr	$\frac{3}{4}$	9	108
4 quarters	1 yard	yd	$\frac{3}{1}$	36	432
5 qr	1 ell English	E E	$\frac{15}{4}$	45	540
3 qr	1 ell Flemish	E fl	$\frac{9}{4}$	27	324
6 qr	1 ell French	E fr	$\frac{18}{4}$	54	648

6. LONG MEASURE			— to ch	miles	leagues
3 barley corns	1 inch	bar in	$\frac{1}{792}$	,00001575	,000005
7,92 inches	1 link	l	$\frac{1}{100}$	,000125	,000041
12 inches	1 foot	ft	$\frac{1}{66}$	,000189	,000063
3 feet	1 yard	yd	$\frac{1}{22}$	,000568	,000189
5½ yds or 25 links	1 rod or pole	r	$\frac{1}{4}$	,00312	,001041
4 rods or 100 links	1 chain	ch	1	,01250	,004166
10 ch or 40 rods	1 furlong	fur	10	,12500	,041666
8 furlongs	1 mile	mi	80	1,0000	,333333
3 miles	1 league	lg	240	3,0000	1,00000
69⅟₂₄ statute mi	1 degree	deg	5546,66	69,0428	23,0142
60 geographic mi	1 degree	deg			
360 degrees	1 circle of the earth		1988434,4	24855,43	8285,14

7. DUODECIMALS			— to feet	inches	bar
12 thirds	1 second	‴ ″	$\frac{1}{144}$	,0833	,250
12 seconds	1 inch or prime	in	$\frac{1}{12}$	1,0000	3,000
12 inches	1 foot	ft	$\frac{1}{1}$	12,000	36,000

8. SQUARE MEASURE			— to sqr ch.	sq yds	sq feet
144 sq seconds	1 square inch	# in	$\frac{1}{627264}$	,00077	,0069
144 inches	1 foot	ft	$\frac{1}{4356}$	,11111	1,00
9 feet	1 yard	yd	$\frac{1}{484}$	1,0000	9,00
30¼ yds or 625 links	1 rod or pole	r	$\frac{1}{16}$	30,250	272,25
16 rods or 10000 l	1 chain	ch	$\frac{1}{1}$	484,00	4356
40 rods or 2½ ch	1 rood or ¼ acre	rd	$\frac{25}{10}$	1210,0	10890
4 roods or 10 ch	1 acre	ac	$\frac{10}{1}$	4840,0	43560
640 acres	1 mile	mi	$\frac{6400}{1}$	3097600	27878400

9. CUBIC MEASURE

			reduced to bush	Wine glns	A&B glns
1728 cub seconds =	1 cubic inch	″in	$\frac{10}{21513}$	= .00433	= .00354
1728 inch	1 foot	ft	$\frac{298}{371}$	7,48052	6.12766
27 feet	1 yard	yd	$\frac{8046}{371}$	201,97404	165.448
40 feet of round or	1 tun	T	$\frac{11920}{371}$	299,22008	245,106
50 feet of hewn timber			$\frac{14900}{371}$	374 02600	306,383
128 ft ie 8 ft long 4 br & 4th	1 cord		$\frac{38144}{371}$	957,50656	784,34

10. DRY MEASURE

			—— to Wine glns	cub feet	cub inches
33,4 cubic inches	1 pint	pt	$\frac{39}{268}$	,0194446	33.4003
2 pints	1 quart	qt	$\frac{78}{268}$	,0388892	67.2006
4 quarts	1 gallon	gln	$\frac{78}{67}$	,1555569	268.8024
2 gallons	1 peck	pk	$\frac{156}{67}$	,3111139	537,6049
4 pecks	1 bushel	bu	$\frac{624}{67}$	1,2444558	2150.4197
8 bushels	1 quarter	qr	$\frac{4992}{67}$	9,9556464	17203.357
5 quarters	1 wey	w	$\frac{24960}{67}$	49,778232	86016.788
2 weys	1 last	la	$\frac{49920}{77}$	99.556464	172033.57
36 bushels	1 chaldron	chn	$\frac{22464}{67}$	44,800408	67415,11

11. WINE MEASURE

			—— to lb Avor of water	cub feet	cub inches
4 gills or 28,875in	1 pint	gl pt	$\frac{167}{160}$	,01671006	28,875
2 pints	1 quart	qt	$\frac{167}{80}$	,03342012	57,75
4 quarts	1 gallon	gln	$\frac{167}{20}$	,1336805	231
			—— to tuns.		
42 gallons	1 tierce	ter	$\frac{501}{3200}$	5,614581	9702
63 gallons	1 hogshead	hhd	$\frac{1503}{6400}$	8,421871	14553
84 gallons	1 puncheon	pn	$\frac{501}{1600}$	11,229162	19404
2 hogsheads	1 pipe or butt	pi	$\frac{1503}{3200}$	16,843743	29106
2 pipes	1 tun	T	$\frac{1503}{1600}$	33,687486	58212

12. ALE & BEER MEASURE			re'd to W.glns	bush	cubic in
35,25 inches =	1 pint	pt =	$\frac{141}{924}$ =	,01635175 =	35,25
2 pints	1 quart	qt	$\frac{282}{924}$	,0327035	70,5
4 quarts	1 gallon	gln	$\frac{282}{231}$	,130814	282
8 gallons	1 firkin of ale	A fir	$\frac{2256}{231}$	1,046512	2256
9 gallons	1 firkin of beer	B fir	$\frac{2538}{231}$	1,177326	2538
2 firkins	1 kilderkin	kil			
2 kilderkins	1 barrel	bar			cubic ft
54 gallons	1 hogshead of beer	hhd	$\frac{15228}{231}$	7,063956	8,8182
3 barrels	1 butt	B			

13. TIME		—	to annual motion miles	sec	diurnal mo deg	miles
	In 1 sec the earth moves		18,948	,041067	,004	,287
60 seconds	1 minute	sec mn	1136.92	2 46402	,25	17 26
60 minutes	1 hours	h	68215 67	147 841	15,	1035 64
24 hours	1 day	d	1637176,	d. g ,9856	360,	24855,4
7 days	1 week	wk	11460233	6 89926		173988
4 weeks	1 month lunar	m.l.	45840934	27.597		695952 04
13 lunar or 12 solar months*		1 yr	595939142	358.76		9047376 52
365d 6h 9mn 14s	1 periodical year		597989090	360		9078605,5
100 years	1 century	cen				

14. CIRCULAR MOTION		---	to mn Time	days	dg of a circle
60 thirds	1 second	‴ ″	,458	,0002818	,000277
60 seconds	1 prime minute	′	24,35	,0169100	,016666
60 minutes	1 degree	°	1460,97	1,014601	1,
30 degrees	1 sign	S	43829,1	30,43803	30,
12 signs or 360°	1 circle of the zodiac		525949,2	365,2564	360,

* * *

* The 12 solar months (constituting what is called the solar year of 365 days) are, January, February, March April, May, June, July, August September, October, November and December "Thirty days *hath September, April*, June and November; February twenty-eight *alone, and all the rest* have thirty-one." Any year that can be divided *by 4 without* a remainder is bissextile or leap year, in which February *has 29 days.*

TABLES of the former Currencies of the United States and those of England, Ireland, &c. with the proportions which each bears to all the others and to Federal Money.

TABLE 15.

Proportions, values or Multipliers.

	reduced to	$	N.Eng. & Virginia.	N.J. Pen. Del.& M.	N.York & N.Caroli.	S.Caro.& Georgia.	Canada & Novasco.	Sterling.	Irish.
N.E. & Vir. currency	1 far=	1/288		xy by					
	4 f= 1 d	1/72	=	5/4 =	4/3 =	7/9 =	5/6 =	3/4 =	13/16
	12 d 1 s	1/6		or	or		or	or	
	20 s 1 £	10/3		add 1/4	add 1/3		sub 1/6	sub 1/4	
N.J. Pn. Del & Mar.	1 far	1/360	4/5		16/15	28/45	2/3	3/5	13/20
	1 d	1/90							
	1 s	2/15	sub		add		sub		
	1 £	8/3	1/5		1/15		1/3		
N. York & N. Carolina.	1 far	1/384	3/4	15/16		7/12	5/8	9/16	39/64
	1 d	1/96	sub	sub					
	1 s	1/8	1/4	1/16					
	1 £	10/4							
S. Carolina & Georgia.	1 far	1/224	9/7	45/28	12/7		15/14	27/28	117/112
	1 d	1/56					add	sub	
	1 s	3/14					1/14	1/28	
	1 £	30/7							
Canada & Novascotia.	1 far	1/240	6/5	3/2	8/5	14/15		9/10	39/40
	1 d	1/60	add	add		sub		sub	sub
	1 s	1/5	1/5	1/2		1/15		1/10	1/40
	1 £	4/1							
Sterling.	1 far	1/216	4/3	5/3	16/9	28/27	10/9		13/12
	1 d	1/54	add			add	add		add
	1 s	2/9	1/3			1/27	1/9		1/12
	1 £	40/9							
Irish.	1 far	1/234	16/13	20/13	64/39	112/117	40/39	12/13	
	1 d	2/117					add	sub	
	1 s	8/39					1/39	1/13	
	1 £	160/39							

Proportions between figures and dimenſions of different kinds; or *values* of one figure or dimenſion compared with another.

Parts of a Circle, reduced to different parts of a *Circle*, *Square*, &c.

Table 16.	To an equal circle's			Side of an equal sq	sd. of sq. inscribd.
	DIAM.	CIRCUM.	AREA		
3,14159 diam. } = 1 Cir.	=,31831	1	$\frac{\text{cir.}}{12.56636}$	,282094	,225079
,31831 cir. 1 diam	1	3,14159	$\frac{\text{di}}{1,273241}$	,886227	,707016
1 di. x ¼ cir. 1 area	$\frac{1,273241}{\text{2d root}}$	$\frac{12,56636217}{\text{2d root}}$	$\frac{1}{1}$	$\frac{1}{\text{2d root}}$	$\frac{,6364}{\text{2d root}}$,

—Square. 1 length x 1 breadth =1 area.
—Triangle. 1 base x ½ perpendicular =1area.
—Polygon 1 circumference x ¼ di = 1 area.
—Ellipsis 1 length x 1 breadth x ,785398 =1 area.
—Any Figure. 1 mean distance round x 1 lg=area or surface.
—A Solid 1 mean area x 1 lg =1 sol X $\frac{1}{\text{,5236 \& 3d root sqd}}$ = an equal Globe's diameter.

Polygons of equal sides & angles.
1 side x 1 side x ,433013 { area of a trigon of 3
1—x1—x1 = area of a tetragon of 4
—x 1,720477——pentagon, 5
—x 2,598076——hexagon, 6
—x 3,633959——heptagon, 7
—x 4,828427——octagon, 8
—x 6,181827——enneagon, 9
—x 7,694209——decagon, 10
—x 9,361——endecagon, 11
—x 11,196——dodecagon, 12

Solids whose sides & heights are equal to those of a cube of which a triangular prism is $\frac{5}{11}=\frac{50}{121}$ of a cylinder.
—square pyramid $\frac{1}{3}=\frac{14}{33}$ cyl.
—triangular pyramid $\frac{13}{90}=\frac{13}{71}$—
—cylinder (of=diam $\frac{11}{14}=\frac{1}{1}$—
—globe $\frac{11}{21}=\frac{14}{21}$—
—cone $\frac{5}{19}=\frac{1}{3}$—
—parabolic conoid $\frac{11}{28}=\frac{1}{2}$—
—hyperbolic conoid $\frac{55}{1[illegible]}=\frac{10}{[illegible]}$—
—parabolic spindle $\frac{44}{105}=\frac{56}{105}$—

Globe. 1 cir. x 1 di=area x ⅙ di.=sol. or cube of di x ,5236=sol.
Spheroid. 1 cir. x 1 di=area x ⅙ di.=sol. or sq of di x lg. x ,5236=sol.

Signs of Figures, Dimensions, &c.

Circle	Ce.	Triangle	tr.	Ellipsis or Oval	el.
Circumference	cir.	Base	bs.	Transverse or lg.	tr.
Diameter	di.	Perpendicular	pr.	Conjugate or br.	cj.
Square	sq.	Hypothenuse	hy.	Cylinder	cyl.
Length	lg.	Polygon	pn.	Solid or Body	sl.
Breadth	br.	Sides	sd. or cir.	Thickness or Depth	th.
Area	ar.	Pyramid	pyr.	Solidity	sol.

The ſpecific gravity of ſeveral kinds of ſubſtance compared with pure water, of which one cubic foot weighs 999,55+ oz. avordupois weight; ſo ,001 ft weighs 1, oz. nearly.

TABLE 17. quant. & qual. of substances.	No. of times heavier than pure water or wt. in oz. avor.	No of times light'r than S. gold.	quant. & qual. of substances	No. of times heavier than pure water or wt. in oz. avr.	No. of times lighter than S. silver.
,001 ft 'malleable platina	=20·170	,39	,001 ft 'solid gun-powder substanc.	=1,745	6,037
,001 ft of pure gold	19·637	,96	,001 ft of allum	1,714	6,146
—standard gold	18·688	1	—ebony	1,117	9,431
—guinea gold	17·793	1,06	—human blood	1,054	9,995
—moidore gold	17·140	1,10	—amber	1,030	10,228
—quicksilver	13·600	1,39	—cow's milk	1,030	10,228
—lead	11·325	1,667	—sea water	1,030	10,228
—fine silver	11·087	1,70	—pure cold water	1,	10,535
—standard silver	10·535	1,72	—red wine	,993	10,609
—rose copper	9·000	2,09	—oil of amber	,978	10,772
—copper	8·843	2,13	—proof spirits	,925	11,389
—plate brass	8·000	2,351	—dry oak	,925	11,389
—steel	7·852	2,40	—olive oil	,913	11,539
—cast brass	7·850	2,40	—loose gunpowder	,872	12,082
—iron	7·645	2,47	—spirit of turpentine	,864	12,193
—block tin	7·321	2,580	—alcohol or pure spir.	,850	12,394
—cast iron	7·135	2,647	—elm and ash	,800	13,169
—lead ore	6·800	2,777	—oil of turpentine	,772	13,646
—copper ore	3·775	5,003	—dry crab tree	,765	13,774
—diamond	3·400	5,555	—ether	,732	14,392
—chrystal glass	3.150	5,996	—white pine	,569	18,515
—white marble	2·707	6,977	—sassafras wood	,482	21,857
—black marble	2·704	6,985	—cork	,240	43,896
—rock chrystal	2·658	7,106	—common air	,0011	8428,
—green glass	2·620	7,309	—inflammable air	,00012	87792,
—clear glass	2·600	7,254	Gravity of the SOLAR SYSTEM.		
—flint stone	2·582	7,313			
—paving —	2·570	7,345			
—cornelian —	2·568	7,355	,001 ft of the sun	11,333	,929
—free —	2·352	8,030	—Mercury	9,166	1,149
—brick	2·000	9,444	—Venus	5,733	1,838
—live sulphur	2·000	9,444	—Earth	4,500	2,341
—nitre	1·900	9,941	—Mars	3,286	3,206
—alabaster	1·875	10,073	—Moon	3,092	3,407
—dry ivory	1·825	10,349	—Jupiter	1,042	10,110
—[illegible]	1·800	10,493	—Saturn	,405	25,948

18. —— VELOCITY OF MOTION. 'In 1 second sound uninterrupted moves 1142 feet, a cannon ball 660. In ¼ sec dense bodies fall 1 ft, in ½ 4ft, in ¾ 9ft, in 1 sec 16ft, &c. Velocity of the Planets in orbit. In 1 sec Mercury moves 30,428 miles, Venus 22,255, the Earth 18,944, Mars 15,312, Jupiter 8,308, Saturn 6,112, Herschel 4,444.'

To expreſs the value of figures by words.

Rule. To the ſimple value of each figure apply the name of the place which it occupies, to this number annex the denomination in which the ſum is given or expreſſed.

Examples.

6
How may 63074,55 $ be expreſſed by words ? A. ſix-
3 0 7 4, $ 5
ty-three thouſand — ſeventy-four Dollars, five-tenths and
,05 5 5 5 5
five hundredths, or five dimes and five cents, or fifty five cts.

How may 90*£*. 16*ſ*. 11*d*. 3*f*. be expreſſed by words ?
9 0 16 s. 11 d. 3 f.
A. ninety — pounds ſixteen ſhillings eleven pence three farthings.

There is a positive relation existing between all different lengths, between different surfaces or areas, between the contents of all kinds of substance, between different measures of time, motion, &c. i.e. every length (whether inches, feet, yds. lines, — or ——) is longer or shorter than any other length, every surface larger or smaller than any other surface, every substance or cubic contents less or greater than any other solidity, every weight heavier or lighter than any other weight, every value more or less than any other value, a certain number of times or parts of a time ; hence we compare,

Length with length ; as, in. ft, yd. with Ch. rods, miles, &c. the diameter of a circle with its circumference, with the side of a square, &c.

Surface with surface ; as, square inches, rods, acres, with square feet, yd, ch, &c. the area of a circle, with that of a square, with a square diameter or circumference. &c.

Substance with substance ; as cubic inches, feet, pints, bushels, with quarts, pecks, gallons &c (whether wet or dry, strong or weak ;) the solidity of a globe, cube, cylinder with that of a cone, prism, spheroid, &c. oz. lb, qr, with lb, pwt, gr, &c. (whether fine or coarse, medical or agricultural,) $ d, c, with £, s, d, &c. (whether real or imaginary, foreign or domestic) A thing or substance, of a certain weight, contains a certain quantity of matter, these [viz. weight & size] determine the specific gravity of that kind of substance, and every thing is worth a certain price (for some purpose or other ;) so the weight, size and value of any substance, might be compared with that of any other, provided this value could be determined ; as the specific gravity and value of gold, silver, copper, brass, with that of water, gold, silver, &c.

Time with time ; as, *h*, mn, sec, with the time in which the earth moves leagues, mi, deg, &c. in its annual and diurnal motion. A thing

To expreſs the value of words of number by figures.

Rule. Place a figure repreſenting the word in the place which the word repreſents, and where a word is wanting place a cipher, (in its ſtead; as for tens, hundreds, &c.) to this number affix the ſign or denomination in which the ſum is given.

Examples.

What figures are equivalent to ſix thouſand — eighty four Dollars & fifty five hundredths? A. 6 0 8 4, 5 5$

th. h. eighty-four, hths.

Expreſs two hundred pounds and thirty four fortieths by figures. A. $200\frac{34}{40}$£, here both units and tens are wanting, ſo 0 0 ſupply their defect.

Expreſs eleven thouſand eleven hundred and eleven by figures. A. 11000, 1100 & 11, = 12111.

• • • • • •

kept in motion, will in a certain time move a certain distance or length, these [viz. time and distance] determine the velocity or swiftness of motion; so the distance, time and motion of any thing in action may be compared with that of any other; as the velocity of sound, of falling bodies, with that of the motion of the earth, planets, &c.

'Numeration, is the art or act of numbering;' and 'Teaches to express numbers by words or characters;' it 'teaches the different value of figures by their different places, and to read or write any sum or number by these ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.' To express any number by words or characters is the act of numbering; and to know how to express any number by words and characters is the art of numbering; so the act or art of expressing any number or value in any form is numeration; whether the number be greater or less than what is esteemed an intiger or unit, whether it be of quantity or quality, and whether the ratio be decimal, duodecimal, or any other; therefore the tables of money, weight, &c. seem more naturally to fall under this head than any other; though the different values are more commonly distinguished different names than different places; as £. s. d instead of pounds, tenths, hundredths, &c. From the preceeding quotations it appears that the expressing of numbers in any form is numeration; as one hundred $ may be expressed by 100 $, by 10 tens, by 10 $. &c. 1 £ by 20 s, by 20 times 12 pence, &c. the diameter of a circle by ,31831 of the cir. &c. 10 tenths by 1, by [illegible] ten &c.

PART 2d. DECIMAL ARITHMETIC,

Is that by which operations are performed with figures whoſe values are determined by their diſtance from unity or the decimal point.

SIMPLE ADDITION,

Is the collecting or adding together of two or more *ſimple* numbers, to ſhow their value in one ſum.

RULE. Place all the given numbers ſo that units may ſtand under units or the decimal points under each other; then, begin with the firſt right hand column, and add one figure to another, to their ſum add the next figure, &c. when the column is added up, place the right hand figure of its ſum under the column and add the reſt to the ſecond column, which add up as before, &c. thus proceed to the laſt column, under which place its whole ſum; and place a decimal point in the ſum total under thoſe above; this ſum will be of the given denomination, & equal in value to all its parts. (See DEM. OF RULES.)

PROOF. Add each column downward, or draw a line between any two of the given numbers and add the ſum of thoſe below the line to thoſe above, and this ſum will be equal to the ſum total; or add two or more of the given figures together, then add the figures of this ſum together, to which add one or more of the given figures, add the figures of this ſum together, &c. until all the given figures are exhauſted, ſet down the remainder, add the ſum total together in the ſame manner, and if the work be right its remainder will be like that of the given numbers; or neglect the nines or threes in the given number and in the ſum total and if the work be right the remainders will be alike.

EXAMPLES.

Addition and Subtraction Table.

Arithmetical Progreſſion or Proportion.*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	1
2	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	2
3	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	3
4	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	4
5	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	5
6	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	6
7	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	7
8	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	8
9	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	9
10	12	13	14	15	16	17	18	19	20	31	22	23	24	25	26	27	28	29	30	10
11	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	11

To learn the Table for addition. Begin at the left upper corner ; ſay, 2 and 2 are 4, 3 and 2 are 5, 4 and 2 are 6, &c. then 2 and 3 are 5, 3 and 3 are 6, 4 and 3 are 7, &c. and *for ſubtraction* ; (it is unneceſſary to go farther than 10 from 19, 9 remains,) ſay 2 from 4 and 2 remains, 3 from 5—2, 4 from 6—2, &c. or 2 from 4—2, 2 from 5—3, 2 from 6—4, &c. then, 3 from 5—2, 3 from 6—3, &c.

* * * *

---

* The numbers, in the table increasing by the addition of an equal number, are in arithmetical progression ; and the relation or proportion they bear to each other is arithmetical ; i.e. the second number 2 is as much more than the first 1, as the 3d is more than the 2d ; and each of the second numbers is as much less than each of the fourth opposit, as each of the third is less than each of the fifth ; and 4 . . 7 : : 5 . . 8 ; and 2 + 3 . . 4 + 5 : : 6 + 7 . . 8 + 9 ; and 4 . . 3 : : 2 . . 1 arithmetically.— The common difference or number +d or −d is called the arithmetical ratio. If two arithmetical series, the one ascending ; as, 1, 2, 3, 4, 5, the other descending ; as - - - - - - 5, 4, 3, 2, 1, whose ratios are equal, be +d together, their sums, 6, 6, 6, 6, 6; will be severally equal ; as,

1, 2, 3, 4,
8, 7, 6, 5,
9, 9, 9, 9,

The numbers which form the series are called the terms, the first and last are called extremes, the rest, means.

E.	\$	d.	c.	m.		\$.d cm
1	1	2	7	7	=	11,277
1	1	3	4	1		11,341
1	1	4	9	3		11,493
1	1	1	6	9		11,169
1	1	6	3	8		11,638
1	1	0	2	5		11,025
6	6	16	31	33	=	67,943

	\$	lb.
Add	4573,25	794,5
together	7294,756	190,
	213,445 proof.	
	394,213—r 8	424,46
		235,09
A.	12475,664—r 8	
		1644,05 A.
proof.	12475,664	
		659,55
		1644,05 proof.

\$
99,99·
17,99·
5,78
4,49·
1,88·
3,99·
2,49·
,67·
,89
1,99·
1,79·
4,99·
2,67·
,98·
,89·
1,79
153,27

In large numbers place a dot opposit each ten, add the —r to the next figure & one for each dot to the next column.

\$.d.c.m.
10,4,5,8
14,0,6,5
9,3,0,6
7,4,
11,0,9,
,0,0,9
52,3,2,8 A.

	£.	s.	d.	cwt.	qrs.	d.
	140	15	11,	4	$3\frac{14}{\text{lb}}$	$9\frac{1}{4}$
	110	12	6,1	2	$2\frac{12}{\text{lb}}$	$2\frac{3}{4}$
	17	18	9,3	4	$1\frac{13}{\text{lb}}$	$4\frac{2}{4}$
	9	4	2,5	3	$1\frac{7}{\text{lb}}$	1
As.	276	49	28,9	13	$7\frac{14}{\text{lb}}$	$16\frac{6}{4}$

Add 22 Eagles, 33 \$, 44d, 55 c, 66 m, together.

22
33
44
55
66
A. 258016 m.

Add 4£. 40ſ. 10d. to 30£. 3ſ. 4d.
A. 34£ 43ſ. 14d.

What is the value of ,00364 \$. ,9636 d. and 10 m? A. 11, cts.

What is the value of 44£. $\frac{6}{20}$ 4*d*. and 7£ $\frac{12}{20}$ 3*d* ?
A. 51£. 18*ſ*. 7*d*.

Add 275 $. to 1204,7 cts. A. 2870,47 dimes.

What is the motion of the earth's equator each hour ?
69251,31 A. =
=The ſum of the diurnal 1035,64 and annual 68215,67 miles. [*Table* 13.]

SIMPLE SUBTRACTION,

IS the taking or ſubtracting of one *ſimple* number from another to find their *difference* : The largeſt given number, is called the *minuend*, the leſs, the *ſubtrahend*, and their difference, the *remainder*.

RULE. Place the leſs given number under the greater, ſo that units may ſtand under units, or the decimal points under each other ; then, find how many the upper right hand figure exceeds the lower ; place this ſum or difference underneath ; but if the lower figure is the largeſt ſubtract it from ten and add this difference to the upper figure, or add ten to the upper figure and ſubtract the lower from this ſum, ſet down the difference, and add one to the next lower figure or diminiſh the upper figure one, and ſubtract as before ; then, place a point under thoſe above, and the difference or anſwer will be in the given denomination. Such a figure as will increaſe the lower figure to the upper is always the true difference. (See DEMONSTRATION OF RULES.)

PROOF. Add the *difference* to the ſubtrahend and their ſum will be equal to the minuend, or ſubtract it *from the minuend* and the difference will be equal *to the ſubtrahend* ; or add all the figures togeth (as in proof of addition) and ſet down the —*rs*. or neglect the 9s or 3s and ſet down the —*rs*; in either caſe the —*r* of the minuend will be equal to thoſe of the other two lines, if the work be right.

EXAMPLES.

	$.	*pounds.*	*E.$ d.c.m.*	*proof.*
From	254,74	499,75	34,7,2,5	—*r*3
Take	142,51	88,76	20,4,6,8,3	‖
				—*r*3
difference	112,23	410,99	14,2,5,6,7	
proof	254,74	88,76		

	£.		$.		*cwt.*	*qr.*	*lb.*
Principal	4796,5	Income	832,	Groſs wt	22	3	$26\frac{14}{16}$
Diſcount	249,	Expence	496,5	Tare	1	2	$20\frac{10}{16}$
Neat	4547,5	Gain	335,5	Neat	21	1	$6\frac{4}{16}$

From 4576 lb. take 2983,56 lb. A. 1592,44lb.

If 10 caſks of allum weigh 33 cwt. 2 qrs. 15 lb. groſs, tare 1 cwt. 1 qr. 10 lb. what is their neat weight?
A. 32 cwt. 1 qr. 5lb.

If the groſs weight of ſugar be 2 cwt. 1 qr. 25 lb. per barrel and tare 21 lb. per barrel; what is the neat weight pr. bar.?
A. 2 cwt. 1 qr. 4lb.

If the premium, of inſuring two thouſand dollars, be 200 $. what is the neat ſum ſecured? A. 18 hundred $.

What is the ſolidity of a ſtone, weighing 55lb. out of water and 44,5 lb. in? A. 10,5 lb. of 27,66 inches each.

How much does the annual, exceed the diurnal motion of the *earth each day?* A. 1612320,77 mi. [Y. 15]

SIMPLE MULTIPLICATION,

Is the increaſing or multiplying of one *ſimple* number by another, to find their *product*.

The largeſt given number is commonly called the *multiplicand*, the leſs, the *multiplier*, and their reſult, the *product*.

Rule. Place the beſt number to multiply by under the other, neglect the ciphers, and multiply the firſt upper figure by the firſt lower one, ſet the right hand figure of this product under its multiplier, and add the reſt to the *product* of the next figure; *which* muſt be found by the firſt multiplier as before: thus proceed with every figure in the multiplier, placing the firſt figure of each product under its (figure of the) multiplier; then add all the products together, and place to the right of this ſum as many ciphers as there are to the right of both the factors; but when there are decimals in one or both of the given numbers, point off as many decimals (from the ſum of the products) as there are in both the factors, and when there are not ſo many figures in the product, ſupply the defect by placing ciphers to the left of it, for the anſwer. (See DEM. OF RULES.)

Proof. Subtract any number from the multiplier, (or ×*nd*) and multiply by this number and by the remainder ſeparately, and the ſum of theſe products will be equal to the total product; or ×*y* the —*rs* of the ×*nd* and ×*r* together, (found as in proof of ÷*n*,) and the —*r* of this *pt* will be equal to that of the total *pt* if the work be right.

The *price* of 1, and whole *quantity* multiplied together *produce the* whole *value*; mean length and *breadth*, produce *area*; the *area* and thickneſs, produce *ſolidity*.

EXAMPLES.

Multiplication and Diviſion Table.

MULTIPLICANDS, or QUOTIENTS. *Quantities* or *breadths.*

Multipliers, or Devisors. Prices of 1, or lengths, &c.

1	2	3	4	5	6	7	8	9	10	11	12	1
2	4	6	8	10	12	14	16	18	20	22	24	2
3	6	9	12	15	18	21	24	27	30	33	36	3
4	8	12	16	20	24	28	32	36	40	44	48	4
5	10	15	20	25	30	35	40	45	50	55	60	5
6	12	18	24	30	36	42	48	54	60	66	72	6
7	14	21	28	35	42	49	56	63	70	77	84	7
8	16	24	32	40	48	56	64	72	80	88	96	8
9	18	27	36	45	54	63	72	81	90	99	108	9
10	20	30	40	50	60	70	80	90	100	110	120	10
11	22	33	44	55	66	77	88	99	110	121	132	11
12	24	36	48	60	72	84	96	108	120	132	144	12

Products or Dividends. Values, or Areas, &c.

To learn the Table. (Conſider the outſide rows of numbers as multiplicands and x*rs* or as ÷*rs* and *qts*; and under one and oppoſit the other, are their ſeveral *pts* and ÷*nds.*) Begin at the left upper corner; ſay, twice two are four, twice 3 are 6, twice 4 are 8, &c. then, thrice 2 are 6, thrice 3 are 9, or 3 twos are 6, 3 threes are 9, 3 fours are 12, &c. then, 4 twos are 8, 4 threes are 12, 4 fours are 16, &c.

	E.	$.	d.	c.	m.		$.		Cents.	
X*nds*	3	4	5	6	7	=	34,567	=	3456,7	price of 1 tun.
X*rs*					2		2		8	No. of tuns.
pts	6	8	10	12	14	=	69,134		27653,6	
							4		[1]	
							276,536	=	27653,6	price of 8 tuns.

What will 2340 glns.		2340	230
coſt at	230 cts. per gln.?	200	200
	702	468000	
	468	70200	30
A.	538200 cts.	538200 proof.	

		proof.	*inches.*
What will	358,5 lb.	3—*r*	24 length
coſt at, (per lb.)	8,5 cts.?	4—*r*	3 breadth.
	179,25	12	72 area.
	2868,0	ǁ	2 thickneſs.
A.	3047,25	3—*r*	144 ſolidity.

	lbs.		
Multiply	275,25	x*nds*	7563,5 rods length.
by	4,31*d.*	x*rs*	400,53 — breadth.

275,25 x ,01 = 2,7525
— x ,3 = 82,575
— x 4, = 1101,00

pts or *As* 1186,3275 *d.*

3025400,0 = 7563,5 x 400,
3781,75 = —— x ,5
* 226,905 = —— x ,03

3029408,655 — area.

	£.	s.	d.	*far.*	T.	cwt.	qr.	lb.	m.	f.	f.
x*y*	40	16	11	3	4	18	3	20	41	6	20 x*nds.*
by				18				32			13 x*rs.*
	320	128	88	24	8	36	6	40			
	40	16	11	3	12	54	9	60			
A.	720	288	198	54	128	576	96	640	533	73	260 *pts.*

. . . .

---

* It is generally best to begin with the higheſt number firſt when the x*r* *consists of a large number of decimals*; and that those to the right *may be neglected if the* product be ſufficiently exact without them.

D

The foregoing *Rule* admits of ſeveral variations; as, when one of the given numbers is 10, 100, &c. or, ,1 ,01 &c. remove the point ſo many places to the right in the other as there are ciphers in 10, 100, &c. or ſo many places to the left as there are decimals in ,1 ,01 &c. for the product required; or, when one of them is 11 or 12, both figures may be uſed as one; or, when one is the product of two or more numbers, firſt multiply (by) one of thoſe numbers, this product by another, &c. the laſt product is the anſwer.

```
xy 6,704            73140     630(=7x90)=630
by    100000,         1,1     2,5=5 x       ,5
   ————————————   ———————                 ————       A.
A. 670400,000     80454,0          5, x    315,0=1575
```

Reduction Descending.

How many farthings are there in

```
        209£. 6s. ?      1£.x 20 x 12 x 4=960 far.
         20 No. of s in 1 £.
     ————
     s. 4186 No. of s. in 209£. 6s.
          12 — pence in 1 s.
     ————
       50232  pence in 4186 s.
           4
    ——————
      200928 far.  A.
```

How many ſeconds are there in 365 days 5h.48m.57s.?

```
                                    24       h in 1 day.
                                ————
                              8765 h. in 1 ſolar year.
24 h x 60 m x 60 s=No. of       60
ſeconds in 1 day.           ————
                            525948 m. in 1.   —
                                60
                           —————
                A.        31556937 ſec. in 1.   —
```

CONTRACTIONS.

As multiplying one number by another is equal to adding either of the numbers together as many times as there are units in the other ; ſo we may uſe a larger or ſmaller multiplier than the given one, and add to, or ſubtract from the product accordingly ; thus, multiply by 10, 100, or 20, 30, 40, or 200, 300, &c. and add to *this product* the product of as many as its multiplier is leſs than the given one, or ſubtract from *it* the product of as many as its multiplier exceeds the given one.

Or when there is 1, 11, &c. in the multiplier, xy by the figures that exceed 1 and add in the figures of the x*nd* in their proper places as often as there are ones in the x*r*.

EXAMPLES.

xy 2755 by 9,9.
10,0
27550,0 pt. of 10,0
275,5 pt. of ,1
A. 27274,5 —*r* 0 proof.

(xy 43210 by 1010
1000
43210000 pt. of 1000
432100 pt. of 10+
A. 43642100)

xy $7,963_{000}$ by 998
15,926 2
A. 7947,074 1000

xy 297 by 297
300 3
89100 300
891 3
A. 88209 —*r* 0 proof

xy 74,9 by $\overset{3}{1097}$
823 90,0 1100
224,7 3
A. 82165,3

xy 325 by 11
A. 3575

2111^{00} x 88
25332 —12
185768 100

11111111111 by 1111
12344444444321

3210312 x 112
359554944

xy 1375 by 91 or 1375^{00} x 91
13750 10 —*d.*

A. 125125

123750 90
1375 1 +*d*

A. 125125 91

. . . .

APPLICATION or PRACTICE.

REDUCTION DESCENDING.

To reduce numbers is to change their form without altering the value.

Sec. 1st. Of Fractions or Compound Numbers.

RULE 1. Multiply the whole number or integer by the diviſor of a fraction, to the product add the dividend, (if there be any) this number placed over the diviſor forms an *irregular fraction*, of the given value, or the number alone is the value of the given ſum in a lower denomination.

Reduce $47\frac{11}{12}$ lbs. to oz. 23,52£. to ſ. & *d.* $16\frac{1}{2}=\frac{33}{2}$ or
12 20 33 halves.

A. $\frac{575}{12}$ or 575 oz. 470,40 *s.* or $\frac{4704}{200}$ £.
12

5644,80 *d.*

Reduce 2 yds. 2 qrs. 2 na. to fractions. 2 2 2
4

8 + 2 = 10 qrs. or $\frac{10}{4}$ yd.
4

40 + 2 = 42 na.

Reduce 75 fur. to rods. 75 x 40 = 3000 A. or $\frac{42}{4}$ qrs.

2. Multiply any two numbers together, and one of thoſe numbers placed under the product ſhows the value of the other; as, $2 \times 4 = 8$, $\frac{8}{2} = 4$ or $\frac{8}{4} = 2$.

Reduce 8 to a fraction whoſe diviſor is 6. $8 \times 6 = \frac{48}{6} = 8$.

3. Place 1 under any whole number, and it forms a fraction of the given value; as, $2 = \frac{2}{1}$, $20 = \frac{20}{1}$, $3{,}2 = \frac{32}{10}$.

4. Multiply all the diviſors of the given fractions together for a *common diviſor*, multiply each dividend into all the diviſors except its own for the dividends required, theſe placed over the common diviſor expreſs the given values ſeverally.

Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, & $\frac{4}{5}$ to thoſe having a common diviſor.

$$\begin{array}{llll}
\div \textit{nds.} \quad \div \textit{rs.} & & & \\
1 \times 3 \times 4 \times 5 = 60 & \frac{1}{2} = \frac{60}{120} & & \frac{60}{120} \\
2 \times 2 \times 4 \times 5 = 80 & & \text{As} \begin{cases} \frac{90}{120} = \frac{3}{4} \\ \frac{96}{120} = \frac{4}{5} \end{cases} & \frac{80}{120} \\
3 \times 2 \times 3 \times 5 = 90 & \frac{2}{3} = \frac{80}{120} & & \frac{90}{120} \\
4 \times 2 \times 3 \times 4 = 96 & & & \frac{96}{120} \\
\hline
 & & & \frac{326}{120}
\end{array}$$

$2 \times 3 \times 4 \times 5 = 120$ Common Diviſor.

Reduce $\frac{7}{10}, \frac{6}{11}, \frac{7}{2}, \frac{5}{4}$. $\left(\frac{7}{10} = \frac{7 \times 11 \times 2 \times 4}{10 \times 11 \times 2 \times 4} =\right) \frac{616}{880}$, $\left(\frac{6 \times 10 \times 8}{11 \times 10 \times 8} =\right) \frac{480}{880}$, $\left(\frac{7}{2} \times 10 \times 11 \times 4 =\right) \frac{3080}{880}$, $\left(\frac{5}{4} \times 10 \times 11 \times 2 =\right) \frac{1100}{880}$ As

5. Multiply all the $\div$*rs* of a *compound fraction* together for the $\div$*r*, and all the $\div$*nds* for the $\div$*nd* of the *ſimple fraction* required Equal $\div$*nds* and $\div$*rs* may be neglected.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of a $ to a ſimple fraction.

$$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{24}{60}\ \$\ \text{A. or } \frac{2}{\underset{3}{1}} \times \frac{\overset{3}{1}}{\underset{4}{1}} \times \frac{\overset{4}{1}}{5} = \frac{2}{5}\ \text{A.}$$

Reduce $\frac{5}{4}$ of $\frac{5}{7}$ of $\frac{8}{12}$ to a ſimple fraction.

$$\frac{4}{1} \times \frac{1}{7} \times \frac{2}{12} = \frac{8}{84}\ \text{A.}$$

Reduce $\frac{7}{8}$ of a penny to the fraction of a pound.

$\frac{7}{8}$ *d.* $= \frac{7}{8}$ of $\frac{1}{12}$ *s.* $= \frac{7}{8}$ of $\frac{1}{12}$ of $\frac{1}{20}$ £. $= \frac{7}{8} \times \frac{1}{12} \times \frac{1}{20} = \frac{7}{1920}$

Reduce $\frac{7}{1920}$ £. to the fraction of a d.

$\frac{7}{1920}$ £. x $\frac{20}{1}$ s. x $\frac{12}{1}$ = $\frac{1680}{1920}$ d. A. = $\frac{7}{8}$

Reduce $\frac{1}{30}$ lb. Troy to pwts.

$\frac{1}{30}$ lb. = $\frac{1}{30}$ of $\frac{12}{1}$ oz. x $\frac{20}{1}$ pwts. = $\frac{24}{3}$ A.

Reduce $\frac{1}{2}$ £ ſterling to the fraction of a $.

pro. $\frac{40}{9}$ x $\frac{1}{2}$ = $\frac{40}{18}$ $ A.

Reduce 4 lbs. Troy to oz. Avor.

$\frac{4}{1}$ x $\frac{2100}{192}$ (pro. by Table 2) = $\frac{8400}{192}$ oz. A.

What is the Av. weight of 3 puncheons pure water ?

$\frac{3}{1}$ x $\frac{5}{16}$ = $\frac{15}{16}$ Tun. A.

6. Multiply the decimal of a higher denomination by as many of the next leſs denomination as make one of the given, and point off to the right as many places as there are in the given decimal, multiply this by the next leſs denomination, point off as before, &c. the numbers at the left of the ſeveral points expreſs the value of the given decimal in known parts of the integer.

What is the value of ,946 £ ?

20

18 s. 11,04 d. A. s. 18,920

12

d. 11,040

How many farthings in ,99 d ?

4

What is the value of 1,6725 cwt. ?

4

£ 3,96 A.

qr. 2,6900

28

1 cwt. 2 qr. 19lb. 5,12 oz. A.

lb. 19,32 x 16

oz. 5,12

To Multiply Fractions. RULE. Reduce the given numbers to ſimple fractions, and x*y* all the ÷*nds* and all the ÷*rs* together for the ÷*nd* and ÷*r* required, as in comp. fr.

What is the ſolidity of a block $1\frac{1}{2}$ ft. long, $\frac{3}{4}$ broad and $\frac{2}{3}$ thick ?
$1\frac{1}{2}=\frac{3}{2}\times\frac{3}{4}=\frac{9}{8}$ area $\times\frac{2}{3}=\frac{18}{24}$ ft. ſolidity. A. or $\frac{3}{1}\times\frac{1}{4}\times\frac{1}{1}=\frac{3}{4}$ ſol.

What is the value of $20\frac{1}{2}$ lb. of Beef at $6\frac{1}{2}$ cts. per lb. ?
$20\frac{1}{2}=\frac{41}{2}\times\frac{13}{2}=\frac{533}{4}$ A $=133\frac{1}{4}$ cts.

Multiply $\frac{3}{4}$ of 9 by $\frac{7}{8}$. $\frac{3}{4}$ of $\frac{9}{1}$ of $\frac{7}{8}=\frac{189}{32}$ A.

To Divide Fractions. RULE. Reduce the given numbers and proceed as in x*n* after the fraction to divide by is inverted ; (becauſe it is the reverſe of x*n*)

Divide $20\frac{1}{2}$ by 4.
$20\frac{1}{2}=\frac{41}{2}$, $4=\frac{4}{1}$ ÷*r* inverted $=\frac{1}{4}\times\frac{41}{2}=\frac{41}{8}$ A.
Divide 24 by 2.
÷*r* inverted $=\frac{1}{2}\times\frac{24}{1}=\frac{24}{2}$ A. $=12$.

What is the breadth of a field $\frac{1}{2}$ a mile long containing one-eighth of a ſquare mile ?
$\frac{1}{2}$ ÷*r* inverted $\frac{2}{1}\times\frac{1}{8}=\frac{2}{8}$ mi. A.

Divide 3 by 9. $\frac{9}{1}$ ÷*r* inverted $\frac{1}{9}\times\frac{3}{1}=\frac{3}{9}$ A $=\frac{1}{3}$

Divide $2\frac{1}{2}$ by $2\frac{1}{2}$. $\frac{2}{5}\times\frac{5}{2}=1$ A. $=\frac{10}{10}$.
What part of 3 is equal to $\frac{2}{3}$ of 2 ?
$\frac{2}{3}\times\frac{2}{1}=\frac{4}{3}\times\frac{1}{3}=\frac{4}{9}$ of 3 A.

Sec. 2d. Established Rates or Prices.

INTEREST is a compenſation, made by the debtor to the creditor for the uſe of money or value received, and uſually eſtimated at a certain ſum per cent. or hundred, agreed on by the parties ; this ſum or rate is eſtabliſhed by law at ſix per cent. per *annum, or* 6\$. (d. lb. and c.) for 100 one year, $=\frac{6}{100}$ *or ,06 of the principal.*
Principal is that for which compenſation is made.

Rate is the ſum per cent. agreed on, or the proportion which the intereſt bears to the principal.

Amount is the principal and intereſt added together.

Simple Intereſt is intereſt on the principal only.

RULE. Multiply the principal by the yearly *rate*, and the product is the intereſt for 1 year; multiply this by the given number of years or parts of a year and the product is the intereſt required.

Or multiply the principal by $\frac{1}{2}$ or ,5 of the ſolar months, or by $\frac{1}{61}$ or ,0164 of the days, and point off two more decimals to the right than there are in both the factors, and the product is the intereſt for the given time at 6 per cent. pr. year.

EXAMPLES.

What is the intereſt of 365 \$. for 4 years at 6 per cent. per year?
365
6 pr. ct. = ,06
yearly int. 21,90 x 4=87,6 \$ A.

What is the intereſt of 500 \$ for 1$\frac{1}{2}$ years at 5 and 6 per cent.?
mo. 500
500
18 x ,5= 9,=$\frac{1}{2}$ ſolar months
,05
45,00, A.
25,00 x 1,5=37,5 \$ A. at 5 per cent.

What is the lawful intereſt of 4375 \$ for 61 days or 2 months?
4375
61 $\frac{61}{1}\times\frac{1}{61}$= 1 ſixth of 1 year.
A. 43,75 \$

What is the intereſt of 44,4 E. for 120 days at 12 pr. cent. per year?
,0164 12=6 x 2
2
,0328
120
44,4 x 3,9360=1,747584 E. A.

What is the intereſt of 345 £. for 2 y. 4 m. and 15 days, at ,06 per year?

rates	,06	,5	,0164
given time	2,	4,	15,
	,12	2,0	,2460

,12
,02 } here the point is removed 2
,00246 } places to the left.

,14246 rate for 2 y. 4 m. 15 d.
345,

49,14870 £. A. or 49 £. 2*s.* 11,688*d.*
20,
2,97400 *s.*

Compound Interest is intereſt on intereſt and principal added together.

Rule. Add the ſimple intereſt to the principal at each ſtated time of payment, and find the intereſt on the amount as on the principal.

What is the compound intereſt on 100 $ at ,06 per year for 4 years?

100
,06

6,00+100=106 - - - amount firſt year.
,06

6,36+106=112,36 do. 2d. yr.
,06

6,7416

119,1016 — 3d yr.
,06

7,146096

A. int. . . . . 26,247696 } —4th y
100,

What is the amount of 240 $ at ,05 per year for 3 years, compound intereſt?

240
,05

12,00+240=252
,05

12,60+252=264,6
,05

13,230

A. 277,83 $

Bank Discount is a compenſation for the uſe of money borrowed of bankers, and is found as intereſt, by multiplying the principal by $\frac{1}{60}$ or ,01666 of the days, including the 3 days of grace which are allowed the borrower after the time of payment expires.

When a note is not redeemed at the time ſpecified it is uſually renewed or eſteemed renewable 2 or 3 days before the time expires; for which the following table of *bank diſcount* is calculated.

What is the discount of 238 $ discounting for 30 days?

,01666
33,=30+3 ds grace

,54978=$\frac{1}{60}$ of the days
238

A 1,30,84764 payable monthly.

What is the interest of 500 $ for 60 days, discount?

01666
63

1,04958
500

A. 5,2479 to be paid every 2m

or 5,00 for 60 days
3=,05 of 60

,25 $ for 3 days.

5,25 A.

Tax is a price or rate, laid on property by and for the ſupport of government, and is found as intereſt.

What is the tax on 1500D at $\frac{3}{10}$ per cent, or 3 per thousand?

1500
3 per thousand or mill = ,003

A. D 4,500

What is the tax on 26300 D at 8 per mill? = ,008

A. 210,400 D

What is the tax on 4040 £ at 1 per cent? A. 40,40 £.

TABLES, OF

Time.	SIMPLE INTEREST at 6 per cent per year calculated for any time under 16				COMPOUND INTEREST at 6 per ct. compounded.		Bank discount from I to 15	Discount at $1\frac{6}{06}$ pr yr. from 1 to 15	premium from $\frac{1}{101}$ to $\frac{15}{115}$	Insurance at any pr. ct under 16	The amount of 1 $ annuity from 1 to 15 y'rs at 6 pr. ct	Time
	Years.	Lunar Months.	Weeks.	Days·	Yearly.	Quarterly.	Months.	Years.	Rates.	Rates.		
1	,06	,0046154	,0011538	,000165	,06 yr.	,015 qr.	,0055	,0566037	,0099009	.0101010	1,	1
2	,12	,0092308	,0023077	,000330	,1236	,030225	,0110302	,1071428	,0196978	0204081	2.06	2
3	,18	,0138462	,0034615	,000495	,191016	,045678	,0165909	,1525425	,0291262	,0309278	3,1836	3
4	,24	,0184616	0046154	,000660	,262476	,061363	,0221821	.1935483	,0384615	.0416666	4.374616	4
5	,30	,0230770	,0057692	,000825	,338225	,077284	,0278041	,2307692	,0476190	.0526315	5,637093	5
6	,36	,0276924	,0069231	,000990	,418519	,093443	,0334571	,2647058	,0566037	,0638297	6,975318	6
7	,42	,0323078	,0080769	,001155	,503630	,109844	,0391411	,2957046	,0654205	,0752687	8.393837	7
8	,48	,0369232	,0092308	,001320	,593848	,126492	,0448564	,3243243	,0740740	,0869565	9,897467	8
9	,54	,0415386	,0103846	,001495	,689478	,143389	,0506031	,3506493	,0825688	,0989010	11,491315	9
10	,60	,0461540	,0115385	,001650	,790847	,160540	,0563814	,375	,0909090	.1111111	13,180794	10
11	,66	,0507694	,0126923	,001815	,898298	,177948	,0621914	,3975903	,0990990	,1235955	14,971642	11
12	,72	,0553848	,0138462	,001980	1,012196	,195618	,0680334	,4186046	,1071428	,1363636	16,86994	12
13	,78	,06=I yr	,015=1q	,002145	1,132998	,213552	,0739076	,4382022	,1150442	,1494252	18,882132	13
14	,84	,0646156	,0161539	,002310	1,260903	,231755	,0798141	,4565217	,1228070	,1627907	21,015064	14
15	.90	,0692310	,0173077	,002475	1,396557	,250231	,0857500	,4736842	,1304444	,1764705	23,275968	15

At the left or right of the above tables any time from 1 to 15 years, months, &c. may be found, and oppoſit the rate is given for that time, under years, months, &c. for intereſt ſimple and compound, diſcount, &c.

EXAMPLES.

What is the ſimple intereſt of 1000 $ for 5 years 3 m 2 w?

,30 table rate for 5 years.
,0161539 do. for 14 w.=3 m 2 w.
,3161539
1000,
$ 316,1539000 A. int. for 5 y 3 m 2 w.

In ſimple intereſt the point may be removed one place to the right to increaſe the rate ten fold, (2 for 100, &c.)

What is the ſimple intereſt of 500 $ for 60 and 600 days?

,0,0,0,99 table rate for 6 days.
,0 0 99 x 500=4.95 $ } As.
,0 99 x 500 49,5 }

What is the compound intereſt of 1200 £. for 12 years, compounded yearly?

1,012196
1200,
$ 1214,635200 int. A.
(2414,6352 - - - - - amount.)

What is the bank diſcount of 200 $ for 1 month?

,0055 - - - - - rate
200,
$ 1,1000 A.

What is the bank diſcount of 3000 £ payable in one year diſcounting every month or 30 days?

,0739076 - - - - rate for 13 lunar m.=1 y.
3000
£ 221,7228000 A. (paid at the expiration, is *equal to the ſum it would have gained for the bank if paid monthly.)*

What is the diſcount of 400 $ paid 3 years before it is due?
diſcount $\frac{6}{106}$ per yr. ,15254 rate nearly.
400

$ 61,01600 A.

What is the commiſſion on 1000 $ at 3 pr. cent. on the purchaſe money? $
premium, ,0291262 x 1000 = 29,1262 A.

Sec. 3d. Geometrical Involution

Is the raiſing of Powers, or continually multiplying a number into itſelf.

Any number is the *root* or firſt power of itſelf.

Rule. Multiply the root or firſt power into or by itſelf, this product is the *ſquare* or 2d power; multiply the ſquare by the root, and the product is the *cube* or 3d power, &c.

To raze the power of fractions; multiply each term into itſelf.

Examples.

Table of Powers, in which every root bears ſuch *proportion** to its ſquare as the ſquare does to the cube, or as the cube to the biquadrate, &c Or the 5th is to the 3d as the 3d is to the 1ſt. or 4th to the 2d.

	Indices			yd	4	5	6	7	8	,9	⅔	length
Roots or	P. 1st	1	2	3	qr 4	5	6	7	8	,9	⅔	br.&th.
Squares	2d	1	4	9	pr 16	25	36	49	64	,81	4/9	area
Cubes	3d	1	8	27	aa 64	125	216	343	512	,729		solidity
Biquadrates	4th	1	16	81	256	625	1296	2401	4096	,6561		
Sursolids	5th	1	32	243	1024	3125	7776	16807	32768	,59049		

. . . .

* Proportion is a relation or comparison existing between two or more numbers; as in the table, each first term is as many times less than the 2d as the second is less than the 3d or as the 3d is less than the 4th. Of any 4 proportionals, (as in the Rule of Three) the first is as many times more or less than the 2d as the 3d is more or less than the 4th; *so such a number* as will bring the 1st to the 2d will bring the *3d to the 4th term.* The number, by which the terms are multiplied *is called the geometrical* ratio. If the terms, of two geometrical

The *indices* in the table, (increaſing by addition are in arithmetical progreſſion, and) ſhow the power of each term, if the firſt of each ſeries be taken as a root; otherwiſe they only ſerve to repreſent thoſe numbers as they ſtand connected, (in arithmetical and geometrical proportion or progreſſion) ſo that the ſum of any two or more indices is the index of the product of the ſame terms in one of the geometrical ſeries; as, if one or more terms be length and breadth, or price and quantity, their indices added together is the index of the area or value. If a board be $\overset{4\ +}{16}$ ft long and $\overset{1\ =}{2}$ broad $\overset{5}{32}$ ft. is the area, (and 5 its index.) Or if one index be ſubtracted from another the remainder is the index of the quotient of the ſame terms in one of the other ſeries. Or half of any index is the index of the ſquare root, and a third is the index of the cube root, &c. of any term repreſented by ſuch index.

What is the value of $\overset{3}{729}$ yds at $\overset{2}{81}$ cts per yd?
$3+2=5$ index of 59049 cts. A.

Divide $\overset{5}{32}$ by $\overset{3}{8}$. $3-5=2$ index of 4 qt. A.

If $\overset{2}{16}$ lb coſt $\overset{1}{4}$ \$ what will $\overset{5}{1024}$ lb coſt?
$1+5=6-2=4$ ind of 256 \$ A.

What is the ſquare root of $\overset{4}{256}$? $4=$twice 2 ind of 16 root. A.

What is the value of a piece of land 216 rods long and 36 broad at 1 \$ per ſquare rod?
$3+2=5$ index of 7776 rods or \$ A.

* * * * * * *

ries, the one ascending the other descending, whose ratios are equal, be multiplied together, their several products will be equal to each other; as,

	1	2	4	8	16	32	16	8	4	2	2 : 4 :: 3 : 6 } ratio 2
	16	8	4	2	1	,25	,5	1	2	4	8 : 4 :: 4 : 2 }
pts.	16	16	16	16	16	8,00	8,0	8	8	8	16 : 16 :: 12 : 12

To form a comparison; there must be two terms, which are sometimes called a couplet, the first term is called the antecedent and the second the consequent; the ratio between these terms is the same of that be*tween any* other antecedent and consequent of a similar quality; as, if *1 yd cost 3 D.* what will 4 yds cost?

ratio 3 as 1 yd : 3 D :: 4 yds : 12 D. A. as 1 is to 3 so is 4 to 12.

$\frac{3}{3}$ $\frac{3}{12}$

Sec. 4th. Reduction of Space

Is the reducing of unknown to known quantities.

A *Triangle* is a figure having three ſides and angles. The longeſt ſide is uſually called the *baſe*, and the ſhorteſt diſtance from *it* to the oppoſit angle or corner the *perpendicular*.

To reduce the contents to known quantities.

Rule. Multiply the length of the baſe by ½ or ,5 of the perpendicular, (or the whole perpendicular by half the baſe,) and the product is the *area* or ſuperficial contents; the area multiplied by the length produces the ſolidity or ſolid contents, in meaſure of the ſame denomination the dimenſions were taken in.*

Any plane may be reſolved into triangles.

Examples.

What is the area of a triangular piece of land, 30,45 chains longeſt ſide and 8 ch half the perpendicular?

30,45 baſe.
8, ½ per.

Ch. 243,60 A. area.

What is the contents of a field containing 4 triangles; each 5,4 acres long, and each perpendicular 2,6 acres?

1 acre being a ſquare of 12,649 rods nearly.

5,4 baſe.
2,6 x ,5 = 1,3 ½ perpendicular.

7,02 area of each triangle.
4

acres 28,08 whole area A.

What is the value, at 24 cts per ft, of a three ſided ſtick of timber 22,5 ft long, and every ſide 2 ft, half the perpendicular ,84? length. ½ per. middle ar. baſe. ſol.

22,5 x ,84 = 18,9 x 2, = 37,8
24, price.

cts. 907,2 A.

Circles and Polygons

Are regular figures, whoſe *centres* are equally diſtant from every *place* in the *circumference* or outſide which is equally diſtant from an angle. Twice the diſtance from the middle of one ſide to the centre is the *diameter*.

To reduce the area to known quantities.

Rule. Multiply half the length of all the ſides or *circumference* by half the *diameter*; (i e. from the centre to the middle of one ſide or circumference;) and the product is the area, (in the given denomination.)

Or xy the ſquare of the diameter of a circle by $\frac{11}{14}$ or ,7854 and the product is the area.

Multiply the diameter of a circle by 3,14159 or $\frac{22}{7}$, and the product is the circumference, which x*d* by ,31831 or $\frac{7}{22}$ produces the diameter.

Examples.

Half the circumference of a circle being 11 and half the diameter 3,5; what is the area?

11,

38,5 area. A.

Three ſides of a hexagon (or ſix-ſided figure) being 48
and half the diameter 14

the area is 672

Required the ſolidity of a Grindſtone, whoſe circumference is 75,43 inches,

& $\frac{1}{4}$ diameter 6 —
area 452,58 —
thickneſs 4 —
ſolidity 1810,32 — A

What is the contents of a pail; whoſe cir. is 9,43 pints

diameter 3, —
4 times the area 28,29 —
4 pints deep=4 x 1 —
ſolidity A. 28,29 —

1 pt being a cube of 3,067 in.

An irregular body being put into a pail of water 10 in. diameter; &if it raiſe the water 6 in. what is the ſolidity of the body?

10 x 10 x ,7854=78,54 pail's area.
6 depth
ſolidity A 471,24 in.

Multiply the circumference of a Globe or Sphere by its diameter, and the product is its area or ſuperficial contents; multiply this by $\frac{1}{6}$ or ,1666 of the diameter, and the product is its ſolidity.

What is the diameter, ſuperficial and ſolid contents of the Earth, (ſuppoſing it a globe,) the circumference on the equator being about 24855 miles?

(24855 x ,31831=)7911,6 diameter, (x ,1666=1318,6)
(24855 x 7911,6=)196652818, area, (x 1318,6=)
259386405804,8 ſolidity. As.

The weight of a piece of metal being 100 lbs. and 97,8 lbs. with a part in clear water; what is the ſolidity of that part?

100 –97,8 =2,2 lb x 27,66 in=(the ſolidity of 1 lb water. *Table* 4) 60,85 in. A.

How much ought to be heaped upon a buſhel of apples, potatoes, &c. if, the buſhel, being 18,5 in. broad, will, beſide its contents of apples, &c. contain 1,3 in. of water, ſand or any fluid on the whole ſurface of the bottom?

18,5 x 18,5 x ,7854=268,91 x 1,3=349,583 inches A. =$\frac{1}{6}$ buſhel nearly.

How many degrees, and miles, has the earth advanced on her 1810th revolution,(ſince the chriſtian era,) it having been 4,11 lunar months ſince the year commenced?

(*Table* 13.) The annual motion of the earth each month is 27,597°+ 113,42367°+ A.
Or the motion is 45840934 mi each month. 188406238,74+A.

How many degrees of longitude has the earth moved ſince noon, being 6,4 hours?

(*T* 13. diurnal motion 15 degrees per h.) 96 deg. A.

Required the weight and ſolidity of a piece of wood, which being put into a pail of water 8 inches in diameter, raiſed the water 4 in. while ſwimming, and being forced under, the water was raiſed 2 in. higher?

8 x 8 x ,7854=50,2656 area of pail x 6=301,59+ ſol. A.
4 1 in. of water weighs ,578 oz.

,678 x=201,0624=116,214 oz. Avor. wt. A.

SIMPLE DIVISION

Is the reverſe of multiplication; and is the dividing of one *ſimple* number by another to find their *quotient.*

The largeſt given number is commonly called the *dividend*, the leſs, the *diviſor*, and their reſult the *quotient.*

The *remainder* is a part of the dividend, leſs than the *diviſor.*

Rule. Find how many times the diviſor is contained in (a ſufficient number of) the left hand figures of the dividend; ſet down this number for the left figure of the quotient; multiply the diviſor by it, and ſubtract the product from the ſaid left hand figures of the dividend; to this *remainder* bring down the next figure of the dividend, and if *it* is ſtill leſs than the diviſor, place a cipher in the quotient and bring down the next figure; then the number of times the diviſor is contained in this ſum is the next figure of the quotient; with which proceed as before, &c. until all the dividend is bro't down, and if there be a remainder place it over the diviſor at the right of the quotient: but if decimals be required in the quotient; annex ciphers for decimals to the remainder or dividend, (until they exceed thoſe in the diviſor, 1, 2, 3, &c.) divide as before as far as neceſſary, and point off as many decimals from the quotient as the decimals in the dividend or remainders exceed thoſe in the diviſor; but *if there be not* ſo many figures in the quotient, *ſupply the defect* by placing ciphers to the left of it. *So there muſt be as many* decimals in the diviſor nd quotient, as there are (uſed) in the dividend or emainders. (See DEM. OF RULES.)

Note. The diviſor is uſually placed at the left and the uotient at the right of the dividend. The ciphers at the ight of the diviſor may be neglected or pointed off, and as any figures at the right of the dividend; theſe figures are ecimals or will be the right hand part of the laſt remainder.

PROOF. Add the remainder and every lower line f the work together; or multiply the diviſor and uotient together and add in the remainder, eith- r of thoſe ſums will be equal to the dividend; or ſe the quotient as the diviſor, and the diviſor will e the quotient, leaving the ſame remainder; or xy he —*rs* of the ÷*r* and qt. together, (found as in roof of +*n*,) and add in *that* of the laſt remainder, nd the —*r* of this ſum will be equal to that of the ÷*nd*, if the work be right.

The whole *value* divided by the *quantity* gives the *value* of , in the quotient; any *contents* divided by one of its *dimenons* gives another *part* or *dimenſion*, according to the capacity f the figure.

EXAMPLES.

If 2300 gallons coſt 475390 cts. what will 1 gallon coſt?

÷r. ÷nd. qt. 1 -- 3= 2

3,00)4753,90(206$\frac{159}{230}$ cts. A. or 23,)4753,9(206,6913 +cA

46 46 23 + —*r*

153 153 4753,9 proof.

138 138

ſt —*r* 15,90 15,9

13,8

roof. 4753,90 by +*n*.

2,10000000

2,07

In the above example the two ciphers at the right of the ÷r being pointed off, to ſhorten the operation, it is requiſite that the ÷*nd* ſhould be equally leſſened; $^{47}\frac{53.96}{23.00}$ is equal to $^{47}\frac{5390}{2300}$ or $^{47}\frac{539000}{230000}$, &c. pointing off 2 figures is equal to ÷*g* by 100, and if both the ÷*r* and ÷*nd* be equally ÷*d* it will not alter their qt.

23 is contained twice in 47, ſo x*y* 23 by 2 and place the pt under 47, ſubtract it therefrom, to the —*r* 1 bring down the next figure, of the ÷*nd* 5, and becauſe the ÷*r* is larger than this ſum 15, place 0 in the qt and bring down the next figure of the ÷*nd* 3, and find how often 23 is contained in 153, and place the No. (of times; viz. 6) in the qt; x*y* 23 by 6 and ſubtract the pt. from 153, to the —*r* 15 bring down 9 or 90, and becauſe the whole ÷*r* is not contained in this ſum there can be no more integers in the qt; then the whole — placed over the whole ÷*r* ſhows what part of 1 the —*r* is equal to; viz. $\frac{1590}{2300}$; ſo the qt is 206 and as near 1 more, (which would make 207,) as 1590 is to 2300, or as 15,9 is to 23.

In the ſecond operation, 9 being brought down, the ſum is found to contain the diviſor ,6 times; ſo x*y* 23 by ,6 and —*t* the pt 13,8 from 15,9, to the —*r* 2,1 bring down 0 in this ſum the ÷*r* is contained ,09 times 23 x*d* by ,09=2,07, ſubtract this from 2,1, and the —*r* is ,03, &c.

```
quantity. whole value. price of 1. whole val. quantity.            proof.
 3 .—  6=           3 .—    6=         3
1820,)8009000,(4400,5)8009000,(1820,                              —r 2
      7280               44005    proof by                   qt's —r 4
      ----               -----       ÷n.                             —
       7290              360850                                      8
       7280              352040                                 90—r 0
       ----              ------                                      —
        1000,0            88100,                                     8
         910,0            88010,                                     ‖
        ------           ------                             ÷nd's —r 8
  —r - - -  90,0= - - - - - - - 90,0
```

Divide 11675 $. between 114 men.

114)11675(102,412 $ qt A.

If a load of wood be 4 ft broad and 4 thick; how long *muſt it be to contain* 1 cord? 4×4=16)128(8 ft A.

to contain 98 ft? 16)98(6,125 ft A.

Every figure, whether a decimal or integer, will hold its proper place in any operation, according to the law of numbers; so the 1st qt figure - - - - - — must occupy the same degree of decimals or integers which that figure - - - - - - - - - - - - - - - — or place of the ÷nd does that stands over (the pt of) the units place of the ÷r - - - - - - - - - - - - — in the first product of the work;

222,)4675,34 (21,060090
4440,

And the 2d qt figure - - - - - - - - - - - - - - — will occupy the same place in decimals or integers which that figure - - - - - - - - - - - - - - — of the ÷nd does that stands over the (pt of the) tenths place of the ÷r - - - - - - - - - - - - - - — in the first product, &c.

2,7),0621 (,023
,054

Also the first qt figure - - - - - - - - - - - - — will be as far remote to the right or left from units as the highest place - - - - - - - - - - - - - — in the ÷nd is distant to the right or left from the highest place of the ÷r - - - - - - - - - - - — except the left part of the ÷r be larger than that of the ÷nd, which moves the qt one place to the right.

23,)227953 (09911,
207000,

2 1 0 1 2 3 —5 4 3 2 1 0 1 2 3 4 5 = 3 2 1 0 1 2 3 4 indices

2 — 5 = 3
323,216) 683927,94326 (2116,0089

2 + 3 = 5 0 = 3 — 3
323,216x2000=646432,=2000x323,216

37495,9,

2 + 2 = 4 1 = 2 — 3
——x 1= 32321,6 = 1 x——

5174,34

2 + 1 = 3 2 = 1 — 3
——x 1= 3232,16 = 1 x——

1942,183

2 + 0 = 2 3 = 0 — 3
——x 6,= 1939,296 = 6, x——

2,8,87260

2 — 3 = 1 6 = 3 + 3
——x ,008= 2,585728 =,008 x——

,3015320

2 — 4 = 2 7 = 4 + 3
——x,0009= ,2908944 = ,0009 x——

— r ,0106376

What is the perpendicular of a triangle containing 45 ft on a baſe 20 feet long ? 20)45(2,25 x 2 = 4,5 ft A.

What is $\frac{1}{4}$ the diameter of a circle 44 in circumference containing 154 ? 44)154(3,5 = $\frac{1}{4}$ diam. A.

What is the breadth of a board containing 27,5 ft 14,6 long ?
146)275(1,88+ ft A.

If a piece of land be 30 ch long and contain 65,257 ch. what is its breadth ?
30)65,257(2,17523333 Ch A.

What is the length of a trapezium whoſe mean breadth is 4 and contents 54 ? 4)54(13$\frac{2}{4}$(ft. in) A.

What is the diameter of an octagon, (having 8 ſides) whoſe contents is 30,72, 2 ſides being 4,8 ? 4,8)30,72(6,4 A.

What is the circumference of a circle, containing 452,58 inches, whoſe diameter is 24 in. ?
24)452,58(18,8575 x 4 = 75,43 in. A.

The area of the end of a cylinder being 210 in. its contents 26250 ; what is its length ?
210)26250(125 in. or 12) 125(10 ft. 5 in A.

£. ſ. d. £. ſ. d. *ft. in. bar. ft. in. bar.*
14)294 280 56(21 20 4 20)492 1720 24(24,6 86 1,2

20) 12) 4)
In 440/ 132 *d.* 746 *ſ.* how many *£. ſ. d.*
(22£ (11ſ (186$\frac{2}{4}$ d. A.

In 192 acres 48 roods 800 rods, how many miles, acres & roods?
640)1920(3 (12 (20 A.
4 48 800

The preceding *Rule* admits of ſeveral variations ; as, when the diviſor is 10, 100, &c. or ,1, ,01, &c. the decimal point *moved as many* places to the left in the diviſor, when it is *10, 100* &c. or as many places to the right as there are deci*mals in* the diviſor, when it is ,1, ,01, &c. forms the quotient required. When the diviſor does not exceed 12, 120, &c. the uſual method is, to place the firſt *quotient-figure* under *its* part of the dividend and if *its* product be leſs than ſaid part, ſuppoſe the remainder to ſtand at the left of the next figure of the dividend, under which place the number of times the diviſor is contained in this ſum; if any figure or ſum be leſs than the diviſor place a cipher in the quotient and conſider the number as a remainder. When the diviſor is the product of two or more numbers, the dividend may be divided by one of thoſe numbers, this quotient by another, &c. the laſt quotient and remainders are the quotient required; to ſind the true —*r*, x*y* the qt and ÷*r* together, ſubtract their pt from the ÷*nd,* the difference is the —*r* ſought.

Divide 4794 by 100. 47,94 A.		÷*e* 4,4 by ,01.
Divide 49,1 by 10000. ,00491 A.		440, A.
2)5475	30)42,27	,5)1042,
qts. 2737½	1,409	2084, As.
,07)49,63	1,1)394	1200)27,6
709,	358,18	,023 As.

Reduction Ascending.

Divide 423155 by 343 = 7 x 7 x 7. A. $1233\frac{236}{343}$

7)423155

7)60450 —*r* 5

7)8635 —*r* 5

qt. 1233 —*r* 4

423155
422919 = 1233 x 343

236 true —*r*.

In 247543 farthings how many £?

4)247543 4 x 12 x 20 = 960 f. = 1 £

12)61885 —*r* 3 f.

20)5157 —*r* 1 d.

257 —*r* 17 s. 1d. 3f = $257\frac{823}{960}$£

CONTRACTIONS.

As dividing one number by another is equal to ſubtracting the diviſor or quotient from the dividend as many times as there are units in the other ; ſo we may uſe a larger or ſmaller quotient than the required one, and add to or ſubtract from this quotient accordingly ; thus xy the ÷r by 10, 100, 20, 30, 200, 3, 4, 5, c and ſubtract the *product* from the dividend or the ÷*nd* from *it* and divide the remainder by the diviſor, and add this *quotient* to the firſt if the pt were ſubtracted from the ÷*nd*, if not ſubtract the laſt qt from the firſt for the qt required

Or divide 1, by the given diviſor and multiply the dividend and this quotient together and the product is the quotient required Or multiply any given diviſor into $\frac{10}{10}$, $\frac{100}{100}$, $\frac{2}{2}$ $\frac{30}{30}$ $\frac{400}{00}$, c then divide this fraction and its quotients by ſuch numbers as will do it without a remainder, and divide the given dividend by it, (ie divide by the upper and xy the quotient by the lower part of it) for the quotient required. The above methods may be inverted for x*n*.

Divide 3537910 by 3545.
3545000 — ÷r x 1000
3545)3537910(1000
7090(2 —t.
7090 ——
qt. 998 A.

Divide 43721,9 by 500
500,)1, . . . (,002
87,4438 qt. A.

Divide 42113,74 by 2,5.
$\frac{25}{10}$x$\frac{100}{100}$=$\frac{250}{10}$=$\frac{10}{4}$
10)42113 74
4
16845,496 qt. A.

Divide 235452437 by 2100
21|00)2354524|37(112000
2352 . . .
2524 37 (120,$\frac{437}{2100}$
252 .
A. qt. 112120,$\frac{437}{2100}$

Divide 432,25 by ,25.
,25)1, . . (4,
qt. 1729,00 A.

Divide 45639,114 by 125.
$\frac{125}{1}$x $\frac{4}{4}$=$\frac{500}{4}$
500 45639,115
91,27823 x 4= A.

```
           23955.....,
   479,1)2395179001,21  (5000000, --

   479,1)     320998,79..(   ┌600,
              28746.,      -t{
                             │
   ---         33538,   (    └ 70,003
               33537,   ---------------
                         4999329,997+
qt 4999329,997 A.      1,79..       qt's —r 7
                       1,4373       ÷r's —r 3
                                        — proof
                       ,3527—r 8 -t from 21=13
                                            ||
                                    ÷nd's —r 4
```

Divide 479136,245 by 66,666+ *=66$\frac{2}{3}$.

```
6 6/10 + = 200/3 )479136,245.
               2395,681225= 7187,043675 qt. A.
```

Divide 4325734 56 by 333$\frac{1}{3}$.

```
333⅓)1,...(    ,003

     1297720,368 qt. A.
```

• • • • • • • • • • • • •

APPLICATION or PRACTICE.

REDUCTION ASCENDING.

To reduce Fractions or Compound Numbers, to higher denominations.

RULE 1. Divide the dividend or given number by the diviſor, or by ſo many of the given denomination as make one of the next higher; the quotient is of the ſame value of the given number, in a higher denomination; this *qt* may be ÷*d* by the next denomination, &c.

Reduce 437531 4ths of a penny to pence.

```
     4)437531

qt. 109382¾ d.   A.
```

```
         20)
Reduce 475793 pwts to lb.

         12)23789 —r 13 pwt.

A. lb.   1982 —r 5 oz.
```

---

* The sign + used at the end of a No as above signifies more or that the No. is too small. And -- signifies less, or that No. is too large.

Reduce $\frac{245}{9}$ ft to its proper terms.

9)245(27$\frac{2}{9}$ ft. A.

Reduce 47558 nails to proper terms.

4)11889 —*r* 2 na

A. yd. 2972 —*r* 1 qr 2

2. Divide the dividend and diviſor of a fraction by ſuch a number as will divide both without a remainder, in the ſame manner divide their quotients, &c. the laſt quotients expreſs the fraction in its ſhorteſt or *higheſt terms.* Or divide the greater number by the leſs, the leſs by the —*r*, the firſt —*r* by the 2d, &c. till nothing remains, the laſt diviſor is the largeſt (number that will divide both without a —*r*, viz.) *Common Diviſor or meaſure.*

Reduce $\frac{1920}{5760}$ lb to its higheſt terms.

8)$\frac{192}{576}$ = ${}^{8})\frac{24}{72}$ = ${}^{3})\frac{3}{9}$ = $\frac{1}{3}$ lb A.

Reduce $\frac{15}{20}$£ to its higheſt terms. 5)$\frac{15}{20}$ = $\frac{3}{4}$ £. A.

Reduce $\frac{96}{144}$ to its higheſt terms. 12)$\frac{96}{144}$($\frac{8}{12}$ $\frac{2}{3}$ A.

Reduce $\frac{444}{777}$ to its higheſt terms

444 777 1   111) $\frac{444}{777}$ = $\frac{4}{7}$ A.

444

333 444 1

333

com. ÷*r* 111)333(3

333

3. Divide the dividend of a fraction by its ÷*r*, and the quotient expreſſes the given value in *Decimals.*

4. Place 1, (with ciphers annexed) under any decimal and reduce it to its higheſt terms, and it expreſſes the given value in a *vulgar fraction.*

Reduce $\frac{3}{4}$ $ to decimals. 4)3,00(,75 $ A. 25)$\frac{75}{100}$ = $\frac{3}{4}$ proof.

Reduce ,25 to a vulgar fraction.

25)$\frac{25}{100}$(= $\frac{1}{4}$ A. 4)1,..(,25 proof.

Reduce 114 s. 11 d. to £.

12)11,0000 20) £.

,9166+ s. + 114 = 114,9166+ (5,7458+ A.

Reduce 10 oz. 18 pwt. 16 gr. to lb.

20) pwt. oz.
24)16,(,6666+ 18=18,6666(,933+ 10,
lb,911111+ A.

Reduce $\frac{17}{25}$ to decimals.

25)17,00(,68 A. 4)$\frac{68}{100}$=$\frac{17}{25}$ proof.

Reduce ,0875 to a vulgar fraction.

25)$\frac{0875}{10000}$($\frac{35}{400}$($\frac{7}{80}$ A. 8),7(,0875 proof.

Reduce 14 s. 11 d. 2 f. to £. 4)2,0

12)11,5

£.
20)14,92+ (0,746+ A.

Reduce 26 quarts 1 pint 2 gills to gallons.

pint. quarts. gallons.
4)2(,5+ 1=,75+ 26=6,6875 A.

To multiply by fractions. Multiply by the upper and ÷*e* by the lower part of the fraction. 4 × $\frac{2}{4}$)=8(2 pt.

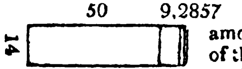
To divide by fractions. Divide by the upper and x*y* (the qt) by the lower part of the fraction. 4 ÷ $\frac{2}{4}$ × 2 = 8 qt.

Multiply 45 by $\frac{3}{5}$. 45 x 3=135 ÷5=27 A.

Divide 27 by $\frac{3}{5}$. 27 x 5=135 ÷3=45 A.

What is the length of a parallelogram, containing 830 rods, whoſe breadth is 14 rods? 14)830(59,2857+ A.

14)830(50
70
14)130(9
126
14) 4,0(,2



The two last figures in the qt 57 amount to 1,1286 inches in the length of the figure, and to ,00798 rd area. ,08 rd in length produces 1,12 area, ,2 length = 2,8 area, 9 length to 126 area, 50 length to 700 area.

Tho' the operation were infinitely extended the ÷nd would not be exhausted, neither would it increase the qt or length of the figure $\frac{1}{4}$ in. more than what is given.

APPLICATION
of the PRECEDING RULES.

Geometrical Evolution,

Being the reverſe of *Involution*, is the extracting of *roots*, or finding the firſt power of a number from any other power.

RULE. Point the given power according to its index; i.e. place a point over units, and if the given power be the 2d, whoſe index is 2, point every 2d figure in integers and decimals, if the given power be the 3d, 4, 5, &c. point every 3d, 4, 5, figure from units: then raiſe a number that ſeems near the *root* to a power whoſe index is one leſs than the given; divide the given number by this power, and divide the difference between the quotient and ſuppoſed root, by the given index, add this quotient to the ſuppoſed root if the root be leſs than the (1ſt) quotient, if not ſubtract it for the *root required.** The firſt figure of any root may be found in the foregoing Table of Powers. It is generally beſt to perform a ſecond operation, uſing 2, 3 or 4 of the firſt figures of the root as the ſuppoſed number; the *root* thus found will be nearer than the firſt, if the firſt be not the true rootor, or *it* will be a proof of the firſt operation.

---

* If we divide any power by its root this quotient by the same root, &c. until the number of divisions be one less than the index of the given power, it is evident that the last quotient will be the root; but, if the given power be divided, in the same manner, by a number less than its root, the last quotient will be larger than the root, and if divided by a number larger than the root, the quotient will be less than the root: so the root is a number between the supposition and quotient, (which is nearer to the former than the latter, the square root excepted;) therefore a suitable part of the difference, added to the smaller or subtracted from the larger of the two extremes, must give a number nearer the root than one of the extremes; and the nearer the *supposition* is to the root, the farther exact the root will be at the first *trial, because the* product of 2 or more equal numbers is larger in *proportion to their sum,* than that of 2 or more unequal numbers; as, $4 \times 4 = 16$, $6 \times 2 = 12$, $4 + 4 = 8 = 2 + 6$, 3)16(5, $+ 3 = 8 \div 2 = 4$ root.

Examples.

What is the length and breadth of a ſquare containing $\overset{2}{625}$ ſquare feet ?

ſuppoſe 22) 625 (28
22
—
given index 2)6(3 + 22 = 25 ft A. proof. 25)625(25

If an army of $\overset{2}{60000}$ men form a column ſquare or nearly ; how many would there be in rank and file ?

ſuppoſe 240) 60000 (250 } A. or 245 rank.
240 } 244+ file.

What is the length of each ſide of a block containing $\overset{3}{1728}$ cubic inches ? 12 in. A.

ſuppoſe 11 x 11 = 121)1728(14
11
——
3)3(1 + 11 = 12 A.

or 10 x 10 = 100)1728(17
10
—
3)7(2 + 10 = 12 A

What is the biquadrate root of 1296 ? Extract the ſquare root twice, or

ſuppoſe 5 x 5 x 5 = 125)$\overset{4}{1296}$(10 – 5 = 5 ÷ 4 = 1 + 5 = 6 A.
6 x 6 x 6 x 6 = proof or ſuppose 6 x 6 x 6)1296(6 proof.

Note. If the *ſuppoſed root* be ſuch that the left qt figure be like *its* left figure, the ſecond figure of the root will be right, if this be near the ſuppoſed root's 2d figure, the 3d figure of the root will be right, at the firſt trial. If the 2 left figures of the *qt* be like thoſe of the ſuppoſed root the 3d and 4th figures will be right, the firſt trial, &c. To find the ſecond figure of the root, by the table of powers ; *—t* the given power of the 1ſt figure of the root, from the left point of the given number ; to the *—r* bring down the next figure, and *÷e* this number by the difference between the power *ſubtracted and the ſame* power of the next larger figure, and t *qt is the 2d figure* ſought. If the firſt figure that is wr *be diminiſhed* one it will be right, generally.

Use of the Proportions in Numeration.

They expreſs the *proportions* exiſting between the Engliſ weights, and meaſures, of different kinds; alſo between th different currencies formerly uſed in the United States, an thoſe of England, Ireland, Canada, &c. between the con tents, dimenſions, &c. of different kinds of figures ſolid an ſuperficial; between the gravities of different kinds of ſub ſtance, &c.

Every proportion repreſents the value of 1 of the preced ing name, in the name or kind written above it; except th left part of the tables, which is the reverſe.

To find the value of one coin, weight, meaſure, dimenſion, &c. in any other.

Rule. Multiply the number to be reduced by the proportion* which it bears to that required, [i.e. by the value of 1 of the given number in that which is required] and the product is the anſwer: the coin, weight, &c. given may be uſually found at the left of the tables and that which is required at the right or atop, and either at the right or left of the given and under or at the left of the required coin, weight, &c. is the proportion, value or multiplier required; invert any multiplier, [or divide by it,] to reduce the coin, weight, &c. written over it, to that at the left.

Examples.

Reduce 6£ [illegible] and 90 *d.* ſterling to dollars.

20 12

120*s* 900*d*

12 (T. 1 *pro.* $\frac{1}{54}$)

1440*d* + 900*d* + 90*d* = 2430*d*

1

54)2430(45 $ A.

• • • • • •

---

* The proportions are given between 1 of the last mentioned denomination at the left and its value in that written over it; when 1 of the *given denomination is larger* than 1 of that which is required, the pr *portion must be as much* larger than unity or 1, which ought to be c *ſidered in finding* proportions.

\$ to £ ſterling.
0,125, or 45÷ $\frac{40}{9}$=
A.

ro. *inverted* $\frac{9}{40}$ *T*.1.)
25£=10£. 2*s*. 6*d*.
)*d*.

O lbs. troy to lbs.
ro. $\frac{576}{700}$ *T*. 2.
$\frac{176}{700}$=82,2857 A.
lb avor to troy.
O

—

O(99,65 lb A.

ſolidity of a caſk
300 lb avor weight

n lb & in. 27,66)
300

A. in. 8298,00
5,92 wine glns. A.

£ N. E. currency
to N. Y. and ſterl-
15)
400 \$
50 £ N. J.
160 £ N. Y.
90 £ Sterl.
As

weight of two cu-
ad? (*T*. 17)
s *oz*. 11,325x,$\frac{2}{001}$
A.

What is the value of 6 lb 4 oz troy of ſtandard ſilver, or ſilver 11 parts fine? (*T*. 2.)

14,056 x 6 = 84,336 \$=6 lb.
1,1713 x 4=4,6852=4 oz.

89,0212 \$ A.

or 6 lb 4 oz=76 oz. x $\frac{991}{846}$=
89,026 A. nearly

How many buſhels of grain will a garner 5 ft ſquare and 4 deep hold?

(T 9. pro. between ft & bu. $\frac{298}{371}$
5 x 5 x 4=100 ft x $\frac{298}{371}$=
80,323 bu. A.
4

pk. 1,292
8

qts. 2,336
or 80 bu. 1 pk. 2,3 q'ts.

What is the avordupois weight of 20 pipes of water, (allowing the caſks to weigh 1½ tuns?) (*T*. 11.)

$\frac{1503}{3200}$ x 20 =9,39375 T
1,5

whole wt. 10,89375

The diameter of a circle being 20 in. what is its cir. & the ſide of an equal ſquare?

3,14 x 20=62,80 circum. A.
,886 x 20=17,720 ſide of a ſq. containing as much as the circle. A. (T. 16.)

COMPOUND ADDITION,
or
SIMPLE ADDITION and DIVISION.

RULE. Add each column of the same denomination together, and divide its sum by so many as make one of the next higher denomination; set down the remainder and add the quotient to the next column, &c. (In the same manner vulgar fractions are added, and their sum divided by the common divisor.)

£	s.	d.	f.
2	19	11	3
1	18	4	1
1	10	11	3
	14	7	2
3	2	2 qts	
	20)63(3	12)35(2	4)9(
	60	24	8
7	3	11	1

$\frac{47}{50}$ common

$\frac{34}{50}$

$\frac{60}{50}$

$50)141(2\frac{41}{50}$ A.

COMPOUND SUBTRACTION,
or
SIMPLE SUBTRACTION and ADDITION.

——. Subtract the lower number from the upper; but where the lower number is the largest, subtract it from [the common divisor or] so many as make one of the next denomination and add the remainder to the upper number, set down this sum for the remainder required, add one to the next lower number and subtract as before.

lb.	oz	pwt	gr.
3	7	10	23
1	$\overset{2}{8}$	$\overset{20}{11}$	$\overset{24}{21}$
1	3	9	2
	7	10	
A 1	10	19	2—r

$21\frac{11}{20}$ $4\frac{1}{4}$

$10\frac{18}{20}$ $2\frac{3}{4}$

1 1

A. $10\frac{13}{20}$ $1\frac{1}{2}$ A.

COMPOUND MULTIPLICATION,
or
SIMPLE MULTIPLICATION, DIVISION and ADDITION.

——. Multiply each number by the multiplier & divide the product by so many as make one of the next denomination, set down the remainder and add the quotient to the next product, which divide as before.

lb.	oz.	pwt.
	9	19
		30
	270	20)570 pts.
	28	28
	12)298	
	24	
A 24	10	10—r

$2\frac{2}{3} \times 4 = 8\frac{8}{3} = (3)8(2\frac{2}{3})10\frac{2}{3}$ A

COMPOUND DIVISION,
or
SIMPLE DIVISION, MULTIPLICATION and ADDITION.

——. Divide each number by the divisor, set down the quotient and multiply the remainder by a number that will reduce it to the next lower denomination, add this sum to the given number of the same denomination and divide as before.

Tuns.	C.	T. C.
4)15	15	(3 18¾ A.

—r3x20=60 cwt.

4)75(18 —r 3

4 q

4)12(3

$2)5\frac{1}{4}(2\frac{1}{4}\frac{1}{2}$ A. or 2 3[qr

$\frac{4}{1}$ —r x 4 + 3 = $7\frac{3}{}$

THE RULE OF THREE,

IN GEOMETRICAL PROPORTION,

ompoſition of multiplication and diviſion, which takes
me from the number of terms given; it is alſo called
ule of proportion,* becauſe the ratio or proportion, exiſt-
e ween the given quantity or cauſe and its required
or effect, is given in numbers of the ſame quality.

LE. When the *anſwer* is to be larger than the *given*
r of the ſame name or quality, multiply *this number* by
rger of the two other given numbers, that are alike in
y; but, when the anſwer ought to be leſs, multiply the
er, that is of the ame name of the anſwer, by the leaſt
two other given numbers, divide the product by the
and the quotient is the number required For proof in-
he queſtion, making the anſwer a part. (*See Dem. of Rules.*)

EXAMPLES.

2 yds. of cloth coſt 3 $ what will 8 yards coſt?
re 3 $ is of the ſame name or quality of the anſwer,
muſt be larger than 3 therefore xy by the largeſt of
milar numbers, viz 8 yds. and divide the product by
her, viz. 2)24(12 $ A.

8 yds coſt 12 $ what will 2 yds coſt?
re the anſwer is to be leſs than the given number of the
name, viz. 12 $ therefore xy by the leaſt of the ſimilar
ers, viz. 2 yds & ÷e by the other i.e. the largeſt of
wo, viz. 8)24(3 $ A [and proof of the firſt.]

To ſtate or arrange numbers Proportionally.

LE. Place the *quantity* or *cauſe* for the firſt term, whoſe
or *effect* is given, (which place) for the ſecond term, and

* * * * * *

ce any quantity or cause under a line, and its price or effect above,
is fraction is the true proportion between any other quantity or
of the same kind and its price or effect, and is the exact price or ef-
I. If any number be xd by the proportion which it bears to that
required, the product is the answer.

the *quantity* or *cauſe* for the third, whoſe *price*, *effect* or anſwer is required for the fourth term, which may be found as before by multiplying the 2nd & 3rd terms together and dividing their product by the 1ſt. or ſuch a *number** as will bring the 1ſt term to the 2d will bring the 3d to the 4th. So the 1ſt and 3d terms are alike in quality, and the 2d and 4th will alſo be ſimilar. As any cauſe is to its effect, ſo is any greater or leſs cauſe, of the ſame kind, to its effect. Or as an effect is to its cauſe, ſo is another effect ſimilar, to *its* cauſe.

If 40 \$ are equal to 9£ ſterling, how many £ are 2700 \$ worth ?
As 40\$: 9£ :: 2700 \$: 607,5£. or 2700 x $\frac{9}{40}$=607,5£
9
———
40)24300(607,5£ A.
Here 40 \$ is the quantity whoſe price (9) is given. And 2700 \$ is the quantity whoſe price is required. 40 \$ and 2700 \$ are alike in quality, and ſo are 9£ & 607,5£.
As 607,5£ : 2700 \$ :: 9£ : 40 \$ proof.

If either the 2d or 3d term be divided by the diviſor and the quotient multiplied by the other the product is the anſwer.

If 3 lbs be worth 12 cts, what is 36 lbs of the ſame, worth?
As 3 lbs : 12 cts : : 36 lbs : 144 cts. or 36 x $\frac{12}{3}$=144cts.
3)12(4 x 36 = 144 cts A.
3)36(12 x 12=144 ——
Here 4 will increaſe 3 to 12, therefore it will increaſe 36 to 144 the A. As 3 x 4 : 12 :: 36 x 4 : 144.

If 4 £ Maſſachuſetts, are equal to 3 £ ſterling, how many ſterling are equal to 100£ Maſſ. ?

. . . . . .

* This NUMBER is the RATIO existing between the 1st & 2nd terms of the statement, also between the 3rd & 4th, and may always be found by ÷g the 2nd term by the 1st. When there are two terms given, one an antecedent or cause, the other its consequent or effect, they show the *relation between* any other antecedent or cause of the same kind, and *its consequent or effect*, so the RATIO, between a cause of one kind and *its effect, is similar to* that between all other causes of the same quality *and their effects*, and VICE VERSA.

As 4 £ Maſs : 3 Sterl :: 100 Maſs : 75 Sterl A.
or 100 x $\frac{3}{4}$=75 A.

If a ſtaff 3 ft long caſt a ſhade 2 ft; how high is that ſteeple or tree whoſe ſhadow, at the ſame time, is 75 ft, the ground being level?
As 2, ſhade : 3, height :: 75, ſhade : 112,5 ft height A.

A buſhel is to a cubic foot as 371 is to 298 (*Table* 9.)

How large muſt a garner be to contain 120 buſhels?
As 298 bsh : 371 ft :: 120 bsh : 149,4 –ft A.
120

bsh. ft 298)44520(149,4 – cubic ft contents.
or 120 x $\frac{371}{298}$=149 4 – ft

If 20 gallons of water weigh 167 lbs; what will 10 hhds. of 63 glns each, weigh? (*Table* 11 *Wine Meas.*)
As 20 glns : 167 lbs :: 10 x 63 glns : 5260$\frac{1}{2}$ lbs A.
630

20)105210(5260$\frac{1}{2}$

If 100 $ gain 6 in 1 year; what will 400 $ gain in 2 years?
As $\overset{\text{prin.}}{100 \times 1}$: $\overset{\text{int.}}{6}$ :: $\overset{\text{prin}}{400 \times 2}$: $\overset{\text{int.}}{48}$ $ A.

When any of the given numbers are vulgar fractions; reduce them to decimals and proceed as before, or reduce the two numbers of the ſame name to thoſe having a common diviſor and uſe their dividends as integers, or invert the 1ſt term and multiply the three terms together and the product will be the anſwer, or 4th term.

If $\frac{3}{4}$ lb coſt 4 dimes, what is 2 $\frac{1}{2}$ lb worth?
As ,75 lb : 4 di. :: 2,5 lb : 13,333di A.
$\frac{3}{4}$)3,00(,75 lb, 2$\frac{1}{2}$=2,5, lb 4

,75)10,0(13,333

or $\frac{3}{4}$, 2$\frac{1}{2}$=$\frac{5}{2}$ x $\frac{2}{2}$=$\frac{10}{4}$ As 3 : 4 di :: 10 : 13$\frac{1}{3}$ di. A.
or $\frac{3}{4}$, *inverted* $\frac{4}{3}$, 2$\frac{1}{2}$=$\frac{5}{2}$, 4d=$\frac{4}{1}$, $\frac{4}{3}$ x $\frac{5}{2}$ x $\frac{4}{1}$=$\frac{80}{6}$=13$\frac{2}{6}$ di

If 3 lb coſt $\frac{2}{7}$ $ what will 2 lb 13 oz coſt ?

$\frac{3}{1}$ lb : $\frac{2}{7}$: : $2\frac{13}{16}$ lb : $\frac{90}{336}$. $\frac{1}{3}\times\frac{2}{7}\times\frac{45}{16}=\frac{90}{336}$ $ A.

What will $\frac{3}{4}$ yd of cloth coſt, if $\frac{1}{2}$ yd coſt $\frac{3}{4}$ $?

As 2 qr : 3 qr $: : 3 qr : $4\frac{1}{2}$ qr $ A. $\frac{2}{1}\times\frac{3}{4}\times\frac{3}{4}=\frac{18}{16}$ A. $=1\frac{1}{8}$ A.

. . . .

APPLICATION.

SEC. 1ſt. *The Double Rule of Three, Conjoin'd and Inverſe* PROPORTION

Are merely ſimple direct proportion, neither can there properly be any other; for, though the terms may be compound, proportion muſt be ſimple; becauſe all the parts of a cauſe,(making the whole,) muſt bear ſuch proportion to all the parts of its effect conjunctly; as all the parts of another ſimilar cauſe does to its whole effect.[*]

Principal or number of agents and time multiplied together, uſually form the whole *cauſe*, and intereſt or length, breadth, thickneſs, &c. the whole *effect*.

When the terms are compoſed of two or more numbers, find the ſimple terms as the caſe may require, and the fourth term as before; and if a part of the 4th be given divide the whole, when found, by the given part, and the quotient is uſually the part required.

If 100 $ in 12 months gain 8 $ and 5 dimes ; what principal in 6 months will gain 30 ?

int. prin. time. int. time prin.

As 8 + 5 d : 100 x 12 :: 30 : 6 x 705,88 $ A.

8,5 : 1200 :: 30 : 4235,3

30 6)

8,5)36000(4235,3 —(705,88+

---

[*] If the proportion be so balanced that each part of the cauſe bears the same proportion to each part of its effect, as each part of another bears to the same part of its effect, the proportion is simple between the *parts severally*, and between the wholes conjunctly; as, if 2 men in 2 *days lay a pavement 10 ft long* and 10 broad; how large would that be *which requires 3 men 3 days to finish*? d's w. ft d's =

As 2 d : 10 br :: 3 d : 15 broad }
As 2 m. : 10 lg :: 3 m : 15 long. } A. or as 4 : 100 :: 9 : 225 square feet.

" If 6 men build a wall 20 ft long, 6 high and 4 thick, in 16 days ; in what time will 24 men build one 200 ft long, 8 high and 6 thick ?"

effect. : cause. :: effect. : cause.
long. br. th men. days. long. br. th. men. days.
As 20 x 6 x 4 : 6 x 16 :: 200 x 8 x 6 : 24 x $_{80}$ A.

480 ft : 96 d's w :: 9600 ſol ft : 1920 day's wk.
96
—— 24)
480)921600 (1920(80 days.

" If 20 men in 12 days, of 15 hours each, can build a wall 100 ft long, 10 ft high and 6 ft thick ; how long a wall, 12 ft high and 5 ft thick, can 40 men build in 15 days of 12 hours each ?" 200 A.

Cause. : effect. :: cause. : effect.
m d h h th m d h b. h. th
As 20x12 x 15 : 100ft lg x 10 x 6 :: 40 x 15x12 : $_{200}$ x 12 x 5
360 hour's work : 6000 cubic ft :: 7200 h. wrk. : 12000 ft
6000
—— 60) A.
3600)43200000(12000 (200

" If when wheat is 6 *s* per buſhel the two-penny loaf weigh 9,6 oz. what ought it to weigh, when wheat is 7,5 *s* per buſhel ?"

As 1 loaf : 6 *s*. x 9,6 oz. :: 1 loaf : 7,5 *s*. x $_{7,68}$ oz. A.

Five £ N. Jer. currency=4 £ N. E. 3 N. E.=4 N. York. 8 N. Y.=5 Canada, and 10 Canada=9 ſterl. How many ſterling are equal to 100 N. Jerſey ?

As 5 x 3 x 8 x 10 : 9 x 5 x 4 x 4 :: 100 N.J. : 60 ſter. A.

SEC 2d. *PREMIUM*, being the ſum to be deducted from any principal, leaving the remainder ſuch as would give that *premium* at the rate agreed on by the parties, may be *found thus ; As 100*, 1000, &c. with its premium added is *to ſaid premium*, ſo is the given ſum to its premium.

If a perſon allows 8 per cent premium for prompt payment of a debt of 2000 $ due 1 year hence; what is the whole ſum allowed, paying nothing on the premium?

prin pre. prin pre. table rate.
As 108 : 8 :: 2000 : 148,148 *D* A. (premi. $\frac{8}{108}$=,074074)
2000

108)16000(148,148+ ,074074 x 2000=148,148D pf.

If a factor receive 1250 $ to purchaſe goods after having deducted his premium of 5 per cent on the purchaſe; what will his premium be?

As 105 : 5 :: 1250 : 59,523 *D* A.

What will be the premium of inſuring 1000£. at 4 per cent on the ſum ſecured only?
As 104 : 4 :: 1000 : 38,46+$ A. or ,0384615x1000=38,4615

What is the premium to be deducted from 500 $ foreign money at 3½ per cent diſcount? 500,
As 103,5 : 3,5 :: 500 : 16,908+ *D* A. [pre ,$\frac{3,5}{103,5}$]

ſo 500 foreign=483,091+

SEC. 3d. *DISCOUNT* is (ſuch) a premium or compenſation, for value received before it is due, (as being taken from the principal, leaves the ſum received ſuch as would, if put at intereſt, amount to the ſum payable at the time of payment;) and is found as premium, thus;

As 100, with the intereſt for the given time added, is to the intereſt of 100, ſo is the given ſum or principal to its diſcount. Or as 100 with the intereſt + d : 100 :: the principal : its preſent worth.

If the diſcount of 100*D* be 6 per year; what is the diſcount on 700 $ for 1 year, and its preſent worth?
As 106 prin : 6 diſc :: 700 prin : 39,6226 diſc A.
6

106)4200(39,6226+ D [$\frac{6}{106}$ pre.]
As 106 prin : 100 preſ. w. :: 700 prin : 660,3773+ D preſ. w. A.

What is the diſcount on 470 $ due 3 years hence at 6 per cent per year ?

As 100+18 (int for 3 y) : 18 :: 470 : 71,78 — $ A.

What is the preſent worth of 500 due 6 months hence and 600 due 2 years hence, at 6 per cent per year diſcount ?

prin. & int. pres. w. prin. & int. pres. w.
As 100+3 (int for 6 m.) : 100 :: 500 : 485,436 }
As 100+12 (int. for 2 y.) : 100 :: 600 : 535,714 } A.

SEC. 4th. *COMMISSION* is an office of truſt, by which *Factors* and *Brokers* are empowered to do buſineſs for other perſons; and *their* compenſations are uſually termed *commiſſion* or *brokerage* and eſtimated at a certain ſum per cent, per mill, or thouſand, on the purchaſe; which ſum is aſcertained as intereſt, or premium.

What is a factors premium, for purchaſing 2200 $ worth of goods, at 2 per cent ? ($\frac{2}{100}$ premi.)

As 100 : 2 :: 2200 : 44 A. or 2200 x ,02=44, $ A.

If a factor receive 2200 $ to purchaſe goods after deducting his premium of 2 per cent on the purchaſe; what will it coſt the owner ?

As 102 : 2 :: 2200 : 43,137 $ A.

($\frac{2}{102}$ prem.=,0196078 table rate.)

A factor receives 1250 $ to purchaſe goods after deducting his premium of 5 per cent on the purchaſe; he pays a broker 1 per cent for aſſiſtance; what is the ſhare of each ?

As 105 : 5 :: 1250 : 59,523 whole pre.
101 : 1 :: 1250 : 12,376 B's ſhare }
47,147 F's ſhare } A.

(5)59,523(11,904 broker's ſhare too ſmall).

What is the difference, between deducting a premium of 4 per cent from 1000 $ and a prem. of 4 per ct on the whole ſum ?

As 100 : 4 :: 1000 : 40, pre. on whole.
As 104 : 4 :: 1000 : 38,46+ do. deduct.
1,53+ difference A.

Sec. 5th. *INSURANCE* is eſtimated an exemption from hazard, or loſs of more than 5 per cent and the price of inſuring, by paying a certain ſum, ſuppoſed to be in proportion to the danger, from the element to which the inſured property is expoſed; this ſum is uſually found as intereſt. "*Policy* is the name given to the inſtrument by which the contract of indemnity is effected between the parties. If the inſured ſuffer any loſs not exceeding 5 per cent he bears it himſelf."

As the *difference* between 100 and its premium is to 100, ſo is the given ſum to that for which a policy is uſually indicted.

"What is the premium of inſuring 2400 $ at 3 per cent?"
As 100 : 3 :: 2400 : 72 *D* A. or 2400 x ,03=72 *D* A.

"For what ſum is a policy taken out to cover 1800 $ when the premium is 10 per cent?"
100 −10=90 As 90 : 100 :: 1800 : 2000 $ A. (pre=⅑ prin.)

"What is the price of inſuring 9358 *D* at 15 per mill?"
x ,015=140,37 9358 A.

Sec. 6th. *FELLOWSHIP* in trade is the agreement of two or more perſons to trade on a joint ſtock; of which each owns a part, and participates proportionally in the loſs or gain.

As the whole ſtock is to the whole gain or loſs, ſo is each perſon's ſhare of the ſtock to his ſhare of the gain or loſs.*

Where the ſtocks are not each employed for the ſame time; multiply each perſon's ſtock by its time of continuance, and uſe the product as that perſon's ſhare of the ſtock; [thus, as the ſum of the products : the whole gain or loſs : : each product : its part of the gain or loſs.]

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* *One person's stock placed over the joint stock shows the proportion between the whole gain or loss and that person's share.*

A, B, C, and D enter into partnerſhip ; A puts into ſtock 1000 $, B 2000, C. 3000, D 4000, with which they gained 1000 ; what is the ſhare of each ?

A's ſtock 1000
B's 2000
C's 3000
D's 4000

			Gain.		Stock		Gain.
whole	10000	:	1000	::	1000	:	100 $ A's
	10000	:	1000	::	2000	:	200 — B's
or as	10	:	1	::	3000	:	300 — C's.
	10	:	1	::	4000	:	400 — D's

or $100 \times \frac{4}{10} = 400 \times \frac{1}{4} = 300 \times \frac{2}{3} = 200 \times \frac{1}{2} = 100$

A, B and C trade in partnerſhip ; A put into ſtock 550 $ for 6 months, B 400 8 months, C 1000 5 months, and they gain 494,52 ; what is the ſhare of each ?

$$\left.\begin{array}{l}\text{A's } 550 \times 6 = 3300 \\ \text{B's } 400 \times 8 = 3200 \\ \text{C's } 1000 \times 5 = 5000\end{array}\right\} : 494{,}52 :: \begin{array}{r}3300 : 141{,}0 \text{ A's} \\ 3200 : 137{,}6 \text{ B's} \\ 5000 : \underline{215{,}9} \text{ C's} \\ 4945{,}5+\end{array}\Big\} \text{A}$$

A and B contracted to do a piece of work for 150 $; to complete which A worked 70 days and B 50 days ; what ſhare of the money did each receive ?

As 70 + 50 : 150 :: 70 : 87,5 A's ſhare −150 = 62,5 B's A.

A, B and C freight a ſhip with wine ; A's wine being 150 pipes, B's 200, and C's 100 ; in a ſtorm the ſeamen threw overboard 80 pipes ; what was the loſs of each ?

$26\frac{6}{9}$ pipes A's loſs, $35\frac{5}{9}$ B's, $17\frac{7}{9}$ C's, A.

SEC. 7th. GENERAL AVERAGE OF *LOSS* AND *GAIN* is the aſcertaining of the mean or average loſs or gain of any party or parties concerned therein, or what proportion thereof any part of the property muſt bear ; how property may be ſold when the ſeller would gain or muſt looſe a certain ſum per cent, mill, &c. and is ſimilar to fellowſhip.

As the whole or any part of the property is to its known *loſs or gain, ſo is* any other part to its required loſs or gain.

[So the quantity, on which the gain or loſs is known, is the firſt term, its gain or loſs the ſecond, and the quantity on which the gain or loſs is required is the third term, and its gain or loſs the fourth.]

The maſter and crew of a loaded ſhip were neceſſitated through danger of ſinking to throw overboard 10 caſks of hard ware and 40 pipes of Madeira wine. When the ſhip arrived at her deſtined port, an average bill was made to adjuſt the loſs among the owners.

Average loſs accrued to ſhip *Fancy* for goods thrown overboard for preſervation of ſhip, freight and cargo.

	dollars			dollars
Ship valued at	12000	must pay at	,25	300
Freight, (wages and victuals deducted)	3000	pays at	25 pr ct	750
A's value of goods	4000	-----	—	1000
B's value of goods	2500	----	"	625
C's value of goods	8500	-----	...	2125
D's 40 pipes of wine valued at	4000	D & E receives of the owner's of ship&goods saved		7500
E's 10 casks of hard ware -----	6000			,4
	40000			

whole stock. D's&E's loss stock. D receives 3000,0
As 40000 : 4000+6000 :: 100 : 25 loss. E ------ 4500

Bought 30 hogſheads of molaſſes for 600 $; if half of it were ſold for 27 per hogſhead and half for 25 per hhd ; what was the average gain per cent ?

	600
paid in duties	20,66
—— for freight	40 78
—— for ſtorage	6,05
—— for inſurance	30,85
	698,33

hhds. dolls.
2)27 + 25(26, x 30=780, value.
698,33 whole coſt.

698,33 : 81,67 gain :: 100 : 11,69+ $ A.

If one lb of coffee coſt 28 cts and be ſold for 31 ; what is the profit on 293 lbs ?

31 −28=3, As 1 lb : 3 :: 293 : 879 cts A.

If cloth be bought for 4,5 doll per yard ; what muſt it be *ſold at to gain* (1 per dime or) 10 cents ?

As 10 : 10+1 :: 4,5 : 4,5+,45=4,95 $ pr yd A.

Sec. 8th. (*BARTER* is the exchanging of one commodity for another; and in order to do this without loſs to either party; find the value of one of the articles to be exchanged; then find how much of the other article this ſum is worth, and the quantity is the anſwer.)

How many gallons of rum at 90 cents per gallon muſt be given in exchange for 50 barrels of flour at 8 $ per barrel?
As ,9 *D* or 90 cts ; 1 gln :: 400 $: 444 $\frac{4}{9}$ glns A.

A having 200 barrels of flour which coſt him 8 $ per barrel, and B ſells Engliſh goods at 20 per cent profit, and would exchange for A's flour; what value of goods in ſterling money ſhould A receive for his flour, allowing B. to make 5 per cent the moſt?

200 x 8=1600, x 1,15=1840, A's amount at 15 per cent.
9
40)16560(414 £ ſterling A.
or as 40 : 9 :: 1840 : 414 £.

Sec. 9th. *TARE* and *TRET* are allowances in weight. Tare is (an allowance for the weight of the box, barrel, bag, &c. which contains the goods ſold by weight. Tret is an allowance ſometimes made for duſt, waſte, &c. of 4 lb in every 104. There is an allowance, called Cloff of 2 lb in every 3 cwt, ſometimes made for the turn of the poize. Groſs weight is the whole weight of the goods, before any allowances are deducted. Neat or net weight is the juſt weight of the goods, after ſuitable allowances are made. To find the neat weight ſubtract the tare; or,

As any quantity is to its weight allowed, ſo is any greater or leſs quantity to its allowance, at the ſame rate. Or as any quantity is to its neat weight ſo is any other quantity to its neat weight.

What is the neat weight of ten hogſheads of tobacco, each weighing 633 lb groſs, tare 89 lb per hhd?

gross 89 lb hhd
As 633 : 544 neat :: 633 x 10 gross : 5440 lb neat A.
or 633 x 10 = 6330 As 633 : 89 :: 633 x 10 : 890 tare
89 x 10 = 890

544 x 10 = 5440 A.

What is the neat weight of 30 barrels of ſigs, each weighing 2 cwt 3 qr groſs, tare 13 lb per cwt ?

112 11 =2 cwt. 3 qr.
13 - 28
gross —— —— bar. lb
As 112 : 99 net :: lb 308 x 30 gross : 8167,5 lb net A =72 cwt 3qr 19,5

What is the neat weight of 14 barrels of potaſh each weighing 400 lb groſs, tare 10 lb per 100 lbs ?

100
10
——
As 100 lb groſs : 90lb neat :: 400lb x 14 groſs : 5040lb neat A

SEC. 10th. STATEMENT OF THE *PROPORTIONS* IN NUMERATION.

As 1 of the given number [or number to be reduced] is to its proportion or value in that required, ſo is the whole given number to its whole value in that required.

Or, as the diviſor of any fraction or proportion is to its dividend, ſo is any given number whoſe name is written oppoſit ſuch fraction or proportion to its value in the denomination written over it ; (1 is the diviſor when the proportion is in decimals.)

Or, as the dividend of any fraction is to its diviſor, ſo is any number of the ſame name of the fraction to its value in that denomination of which the fraction is the value of 1.

Proportions.			What is the value of, or reduce.	Answers or Values.	
As 1 £ ster	is to 4,4444 \$	so is	400 £ sterling	to 1777.776 \$	A.
— 1 ----	: $4\frac{4}{9}$ ---	::	90 ——	: 400. ---	—
— 9 ------	: 40 ---	::	740 ——	: $3288\frac{8}{9}$ -	—
— 40	: 9	::	400 \$	: 90 £ ster	—
— 20	: 1	::	475 shillings	: 23.75 --	—
— 1 shilling	: 12 pence	::	150 shillings	: 1800 pence	—
— 700 lb troy	: 576 lb avor	::	1400 lb troy	: 1152lb avor	—
— *846 oz silv.*	: 991 \$	::	120 oz troy silver	: 140,56 \$	—
— *9 cwt avor*	: 1225 lb troy	::	27, cwt avor	: 3675 lb troy	—

1 is to 27,664 so is 300 lb avor wt. of water to its sol. 8299,2 in. A

4 E. Fr : 18 ft :: 48 E. French : 216 ft.

371 cubic ft : 298 bushels :: 1113 cubic ft : 894 bushels

1 cubic ft : 7,48 W. glns :: 20 —— : 149.6 W. glns

1 —— : 6,1276 A B. glns. :: 20 —— : 122,552 A.B glns.

67 bushels : 624 glns W. :: 4.69 bushels : 43 68 W. glns.

1 —— : 2150,4197 in :: 12 —— : 25805,0364 cubic in.

160 : 167 :: 1820 pints of water : its weight 1899 625 lbs avor.

1 tun wine : 33,687 cub ft :: 90 tuns wine : 3031,83 cubic ft

231 : 282 :: 693 A.&B. glns : 846 wine glns.

1 : 282 :: 40 —— : 11280 cubic inches.

1 : 68215,67 :: 3 hours : 204647,01mi. the Earth's anl. mo in 3 h.

1 : ,9856 :: 30 days : 29,568 deg. the Earth's anl. mo. in 30d.

1 : 15 :: 20 hours : 300 deg the Earth's diurnal motion in 20h.

1 : 17,26 :: 5 minutes : 86.3 miles the Equator's diurnal mo. in 5 m

1 : 24 35 :: 6′ anl. mo. : 146,1 mn. time of the Earth's mo. 6′ in orbit

3 : 10 :: 360 £ N E &Vir. Currency : 1200 dollars

4 : 5 :: 60 —— : 75 £ N.J.&c. cur.

4 : 3 :: 700 —— : 525 --- sterling

3 : 8 :: 834 --- N.J. &c. : 2224 dollars

15 : 2 :: 75 shillings --- : 10 ——

4 : 10 :: 77,5 £ N. C. : 193,75——

4 : 3 :: 440 —— : 330 £ N.E. &c.

7 : 30 :: 770 £ S. C. : 3300 dollars

1 : 4 :: 75,5 --- Canada : 302 ——

9 : 40 :: 270 --- sterling : 1200 ——

39 : 160 :: 19,5 --- Irish : 80,0 ——

1 : ,31831 :: the cir. of a circle (10) : its diameter (3,1831)

as 22 : 7, :: the cir. of a circle (10) : its diameter (3,1818+)

The circumference of a circle is larger than the diameter; but, to ace 1 cir to 1 diam, the xr must be less than ÷r; and the same to the solidity of 1 prism, pyramid. &c. from that of the cube (or par-lopipedon of the same sides) and to find the number of the less in the ater, the reverse.

part of the *proportions* in table 16, depending on each other for *nitude, are not so well* adapted to a statement in the rule of three *e former;* and each being similar to a separate rule the operation *e more easily performed* by the table.

As 1 is to 3,14159 so is the di. of a circle (20) to its circumfer.(62.8318) A.
As 1 : ,7854 :: the sqr of a circle's di. (3x3) : its area (7 0686) A.
As 1 : ,433013 :: the sqr of 1 side of trigon (2 x 2) : its area (1,732052) A.
As 1 : 1,720477 :: —— of a pentagon (2 x 2) : its area (6 881908) A.
As 11 : 5 :: the sol. of a cube (55) : the sol of a triangular prism (25) A.
As 3 : 1 :: —— (361) : —— sq pyramid (120,333) A.
As 21 : 11 :: —— (84) : —— a globe (44) A.
As 3 : 1 :: —— cylinder (27) : —— a cone (12.333) A.
As 1 : ,5236 :: the cube of diameter (2x2x2) : —— a globe (4,1888) A.

As ,001 ft of lead : 11,325 oz avor. :: 3 feet of lead : its weight
33975 oz avor. A.

As ,001 : 10,535 :: ,02 ft of standard silver : its weight 210 7 oz
avor. value =246,79291 Dollars

As [,001 ft=] ,059844 pt : ,925 oz :: 2 pints proof spirits : its weight
30,913 oz avor.* A.

As 2,6 oz : ,001 ft :: 260 oz of clear glass : its bulk or solidity
,1 cubic ft. A.

As 1 sec. : 1142 ft :: 6 sec. : the distance sound moves in 6 sec.
6852 ft. A.

As 1 x 1 qr sec. : 1 ft :: 6 qr sec. x 6 : the distance bodies fall
(in 1½ sec.) 36 ft. A.

As 1 sec. : ,216 mi :: 8 (vibrations of a 39.2 inch pendulum) : the distance sound moves in 8 sec. 1,728 miles. A.

As 1 sec. : 30,459 mi :: 30 sec. : the distance Mercury moves
(in 30 sec) 913,77 miles. A.

As 1 sec. : 22,295 mi :: 30 sec. : the distance Venus moves
(in 30 sec.) 668,85 miles. A.

As 1 sec. : 18,948 mi :: 30 sec. : the dist. the Earth moves
(in 30 sec.) 568,44 miles. A.

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* If a quart (or 2 pts) wine measure of spirits weigh 30,9+ oz avor the spirits are proof, and the less spirituous liquors weigh the stronger, and the heavier, the weaker the liquor is, till it is reduced to water, a quart of which weighs 33,4 oz avor.

Sect. 11th. The *POWER* by which effects are produced s the ſtrength, weight or moving cauſe applied to produce uch effects.

As any power is to its effect produced, ſo is any other ſim- lar power to the effect it is capable of producing. Or, as an ffect is to its power, ſo is another to its power, of a ſimilar ind.

If 20lb. power be ſo placed as to raiſe 240lb; how much ould 150lb. raiſe in ſimilar circumſtances? 1800 A.
As 20lb. power : 240 effect :: 150lb. power : 1800lb. ffect.

If a given ſtrength draw or impel 1 tun up a hill whoſe aſcent is 12°; what weight will double the ſtrength draw up a hill whoſe aſcent is 15°, at that rate?
As 1 power : 12° x 1 tun :: 2 powers : 15° x 1,6 tun A.

"If a body weighing 30lb, be impelled by ſuch a force as to ſend it 20 rods in a ſecond; with what velocity would a body weighing 12lb. move, if it were impelled by the ſame force?"
As 1 power : 30lb. x 20rods :: 1 power : 12lb. x 50rds. A.

When the ſtrength or weight of the Power is not equal to that of the effect, its action or motion muſt in proportion exceed; therefore,

As the velocity* of the effect is to the velocity of the power, ſo is the weight or ſtrength of the power to that of the effect; (the friction abated)

So the velocity and weight or ſtrength of the power (multiplied together) is equal to the velocity and weight or ſtrength of the effect (multiplied together,) while one is juſt ſufficient to balance the other; therefore, to put the power ſo ſuſpend-

* * * * * * *

[*] Velocity is here conſidered as the diſtance which the effect will move, *compared with the diſtance the power moves in the same time; but their real speed or* swiftness, depends on the over balance of power

ed, in motion, there muſt be as much added to it as will balance the friction,* and ſpeed of the motion; therefore the loſs of power is always equal to the gain of ſtrength.

NOTE. The velocity of the power, is to that of the effect, as the distance between the power and center of motion, is to the distance between the point where the effect is produced and the centre of motion. Or, as the circumference (of the circle) described by the point where the power is applied, is to that described by the point where the effect is produced, so is the velocity of the power to that of the effect; and so is the weight or strength of the effect to that of the power, the friction abated.

If a weight of 160 lb. reſt on the end of a lever 10 ft. from the prop or fulcrum; what weight will it balance on the other end of the lever 1 ft. from the prop, excluſive of what the lever balances?

The upper corner of the prop is the centre of motion, the lever (11 ft.) is the two semidiameters of the circles which might be described by the ends of the lever or weight and power moved round the centre, 10 ft. is the distance between the power and centre.

As 1 ft. : 10 ft. :: 160 lb. : 1600 lb. A.
Or, 1 velocity : 160 power :: 10 velocity : 1600 effect.

As 10 : 1 :: 1600 : 160 lb. power required to balance 1600 lb. effect, velocity 10 to 1.

As 160 : 1 :: 1600 : 10 ft. diſtance between the power and centre of motion.

As 1600 : 10 :: 160 : 1 ft. diſtance between the effect and centre of motion.

160 lbs. will balance 160 lb. both equally distant from the centre of motion, this performed ten times at the expence of the power will balance the whole 1600 lb. in separate quantities, making the loss of power equal to the gain of strength, or the cause equal to its effect, for if the substance of one is not equal to the other, the distance restores the equality.

At what diſtance from each other muſt pound notches be made on a *Steelyard*, whoſe poize or power is to weigh 2 lb.

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* *If the power* be so suspended, as to be as easily moved one way as *the other, the addition* of power to create a motion is only the balance *of half the friction*, and whole velocity or swiftness; this equipoise is always *supposed* to be the situation of the power when motionless.

and the diſtance, between the point on which the weight is to hang and the centre of motion (or point on which the ſteelyard is to hang,) to be ,4 of an inch, on a line ſtrait with the beam?

As 2 lb. power : ,4 in. velocity :: 1 lb. power : ,2 in. A.
The firſt notch depends on the weight of the ſteelyard.

If a mechanic would make a windlaſs in ſuch a manner as that 1 lb. applied to the circumference of the wheel ſhould balance 10 lb. at that of the axle which is to be 3,14+ ft. or 1 ft. in diameter; how large muſt the wheel be?

cir. cir.
As 1 lb. p. : 3,14 ft. velocity :: 10 lb. p. : 31,4 ft. A.
Or as 1 lb. p. : 10 lb effect :: 1 ft. di. : 10 ft. di. A.

So the velocity of the ascending Weight (or rope) is to that of the descending power as 1 : 10; or as 1 rope to the other.

How much will a man weighing 160 lb. balance on a ſingle tackle or rope, employing 2 pulleys?

As 1 vel. : 2 vel. :: 160 lb. p. : 320 lb. A.

Here the weight's ascent is to the power's descent as 1 : 2 or = a windlass whose wheel is double the axle; 4 pulleys = a windlass whose wheel is 4 times as large as the axle, &c. except the friction is greater than that of a windlass.

What weight may be raiſed by a Screw, or how hard will one preſs, whoſe ſpiral thread at each turn is 1 inch perpendicularly aſunder, or which will raiſe 1 inch in performing a circle; the lever by which it is turned 30 inches long, and the power applied to each end 10 lb. (or 20 lb. to one end;) allowing nothing for the friction, which is eſtimated on a medium at $\frac{1}{3}$ of the power; but depends on the curvature and ſize of the ſcrew, the diſtance through which it paſſes, &c.?

As 1 in. vel. : 30 x 3,1416 in. vel. :: 20 lb. p. : 1884,96 lb. A.

Here the velocity of the weight is to that of the power as 1 inch (the distance the weight raises) is to 94,248 inches (the circumference of the circle the power describes or distance it moves to raise the weight 1 inch:) so the circle of the power is as the wheel of a windless, & the distance between the threads as the axle, = 94+ pulleys.

Sec. 12th. *SIMILAR FIGURES* are figures of ſimilar ſhapes, whoſe ſimilar ſides are all proportional to each other, and equal in number, and whoſe angles are the ſame.

As one ſide or dimenſion is to the ſimilar ſide or dimenſion, of any other ſimilar figure ſo is any other dimenſion to its ſimilar dimenſion, and ſo is the ſum of all the dimenſions of the firſt to that of the ſecond, &c.

As the ſquare of one dimenſion of any figure is to the area of that figure, ſo is the ſquare of a ſimilar dimenſion of any other ſimilar figure to its area. The ſame of (the product of) two dimenſions, &c.

As the cube of one dimenſion of any ſolid is to the cube of the ſame dimenſion of any other ſimilar ſolid, ſo is the ſolidity of the firſt to that of the ſecond.

Note. The sum of the squares or cubes, of two or more dimensions, or the square or cube of the sum of two or more dimenſions, may be used in all respects instead of the square or cube in the above methods.

If one ſide of a hexagon be 1 ft and its area 2,6; what is the area of that whoſe ſide is 3 ft ?

As 1 x 1 : 2,6 :: 3 x 3 : 23,4 ft. A.
By Note 1 x 1+1 x 1 : 2,6 :: 3 x 3+3 x 3 : 23,4.

If one ſide of a block be 2 ft. and another ſide 3 ft. and its contents 10 ft; what is the contents of one in the ſame ſhape whoſe ſimilar ſides are 4 ft. and 6 ft ?

As 2 x 2 x 2 : 10 : : 4 x 4 x 4 : 80 ft. A.
As 2 : 3 :: 4 : 6 proportional ſide 2+3=5.
As 5 x 5 x 5 : 10 :: (4+6=) 10 x 10 x 10 : 80 ft. A.

The diameter of a circle being 4 and its area 12,566368; what is the area of that whoſe diameter is 8 inches?

As 4 x 4 : 12,566 +:: 8 x 8 in : 50,264 +inches. A.
Or as 1 x 1 : ,7854 :: 8 x 8 : 50,2656 inches. A.

PART 3d. *COMPOUND ARITHMETIC*,

IS that by which operations are performed with numbers whoſe values are determined by the diviſion and ſubdiviſion of the integer or unit, and is a compoſition of the preceding RULES.

COMPOUND ADDITION,

IS the adding together of two or more *compound* numbers, to ſhow their value in one ſum.

RULE. Place all the given numbers of the ſame denomination under each other; then, add the numbers of the firſt denomination together, and divide its ſum by ſo many as make one of the next denomination; ſet the remainder under its column, and add the quotient to the next column; which add together and divide as before; thus proceed to the laſt denomination, under which, place its whole ſum.

EXAMPLES.

Sterling Money.				Troy Weight.			
£.	*s.*	*d.*	*f.*	*lb.*	*oz.*	*pwt.*	*gr.*
706	19	11	3	347	11	19	23
225	17	9	2	154	10	17	9
47	13	10	3	63	11	0	10
65	10	11	2	24	0	10	20
3	3	2		40	11	2	10
	20)62 3	12)43(3	4)10(2				
	60	36	8	631	9	11	0 A
1046	2	7	2 A	128	10	13	16
1046	2	7	2 pf.	631	9	11	0 pf

Apothecaries Weight.

lb.	*oz.*	*dr.*	*ſcr.*	*gr.*
247	11	7	2	19
214	9	6	1	10
205	10	0	0	18
27	7	4	2	16
696	3	3	2	3

Avordupois Weight.

tun.	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
225	19	3	27	15	15
33	17	2	14	9	9
353	14	1	0	5	5
46	0	0	25	10	10
659	12	0	12	9	7

Cloth Meaſure.

yd.	*qr.*	*na.*	*in.*	*E.E.*	*qr.*
223	3	3	$1\frac{1}{4}$	34	4
33	2	2	$0\frac{2}{4}$	20	3
17	1	1	1	40	2
73	0	0	$1\frac{1}{4}$	9	0
347	3	3	$1\frac{3}{4}$	104	4

Long Meaſure.

circ.	*deg.*	*ml.*	*fr.*	*ch.*	*rod.*	*yd.*	*ft.*	*in.*	*bar*
32	395	$68\frac{1}{3}$	7	9	3	$4\frac{1}{2}$	2	11	2
40	300	51	4	2	1	0	1	6	1
22	200	30	3	0	0	3	0	7	0
9	36	22	2	7	1	2	2	4	2
105	213	$33\frac{2}{3}$	1	9	3	$0\frac{1}{2}$	1	5	2

Duodecimals.

ft.	*in.*	*″.*	*‴.*	*ft.*	*′.*	*″.*
34	11	11	11	32	11	11
24	9	9	9	50	0	4
97	4	4	4	44	10	6
75	6	6	6	30	8	9
				158	7	6

Square Meaſure.

acre.	*ch.*	*rod.*	*yd.*	*ft.*	*in.*	*″.*	*‴.*
639	9	15	$29\frac{1}{4}$	8	143	143	143
214	7	0	20	4	30	75	54
400	7	6	30	7	96	94	94
96	5	0	4	0	95	70	80

Cubic Meaſure.

tun.	*ft*	*round in.*	*″*	*cord.*	*ft.*
375	39	1727	1727	21	127
397	27	900	900	10	100
429	30	600	600	0	33
228	37	429	347	4	30

Dry Meaſure.

laſt.	*w.*	*qr.*	*bu.*	*pk.*	*gln.*	*qt.*	*pt.*	*in.*
375	1	4	7	3	1	3	1	32,6
120	0	3	4	1	0	2	$0\frac{1}{4}$	30,4
179	0	4	0	2	1	0	$1\frac{1}{2}$	7,
95	1	3	2	0	1	2	0	20,4
						1	1	23,2

Ale and Beer Meaſure.

but.	*bar.*	*kil*	*fir.*	*gln.*	*qt.*	*qt.*	*in.*
43	2	1	1	7	3	1	34,2
73	1	0	0	4	3	2	32,0
99	2	1	1	6	2	0	1,4
46	0	0	1	5	0	1	9,2

Wine Meaſure.

tun.	*pipe.*	*hhd.*	*gln.*	*qt.*	*pt.*	*in.*
147	1	1	62	3	1	27,8
279	0	0	60	2	0	20,4
710	1	1	9	2	1	11,2
431	0	1	27	0	1	9,1

Time. Motion.

Cen yr.m wk.d.h. mn.		Signs	°	′	″	‴	
18,10 4 3 4 18½	☉ is now in ♉	1	27	51	53	22+	from Aries first point.
In 18,10 9 2 2 6 30	— will be in ♎	4	8	37	19	3+	from ♉.
— 18,11 1 0 6 18 30	— will be in ♒	4	8	37	19	3+	from ♎
— 18.11 4 3 4 18½	— will be in ♉	3	11	31	3	46+	from ♒

Between the above dates the Earth will have performed } 1 circle (1 26 37 35 14+) { lacking 1° 14′ 17″

occasioned by the computation of time being 1,2564 day } . . . . (=1 14 17 56+) less than the

true periodical year, if this be +d the sun will be } . . . . . (1 27 51 53 10+ from Aries) in

the same situation it is now in (except the small loss of $\frac{1}{5}''$) having no respect to the difference of time in which the sun enters the signs.

COMPOUND SUBTRACTION,

IS the ſubtracting of one *compound* number from another, to find their difference.

RULE. Place each denomination of the leſs number, under the ſame one of the greater; ſubtract the firſt lower number from the firſt upper one, and place the difference underneath; but if the lower number be the largeſt, ſubtract it from ſo many as make one of the next denomination; to this remainder add the upper number, which ſum place underneath; then either add one to the lower number or take one from the upper, and proceed as before.

EXAMPLES.

Sterling Money.

£.	s.	d.	f.
147	19	7	3
94	19	11	2
52	19	8	1
94	19	11	2

S.	d.	f.
29	3	2
17	4	3
11	10	3
29	3	2

Troy Weight.

lb.	oz	pwt.	gr.
120	11	10	23
49	2	19	14
71	8	11	9
120	11	10	23

oz.	pwt.
27	14
26	19
0	15 A.
27	14 pf.

Apothecaries Weight.

lb.	oz.	dr.	ſc.	gr.
479	3	7	2	16
64	8	4	1	17
414	7	3	0	19

ſcr.	gr.
40	18
37	16
3	2

Avordupois Weight.

	tun.	cwt.	qr.	lb.	oz.	dr.
groſs	420	16	2	20	15	14
tare	99	19	1	20	10	15
neat	320	17	1	0	4	15 A

Cloth Meaſure.

yd.	qr.	na.	in.
740	3	2	$1\frac{3}{4}$
74	2	3	2
			$1\frac{1}{3}$

E.E.	qr.
75	4
26	2

Long Meaſure.

deg.	mi.	fur.	ch.	l.	in.
359	40	4	7	75	6,97
240	47	7	3	99	7,91

Duodecimals.

ft.	in.	″	‴	⁗
140	10	3	4	11
79	9	6	4	3

ft.	in.	″
40	11	2
39	7	3
1	3	11

Square Meaſure.

acre.	rood.	rod.	yd	ft.	in.	
475	3	27	20	7	130	14
234	2	16	$23\frac{1}{4}$	3	70	7

Cubic Meaſure.

cord.	ft.	in
175	140	747
29	29	974

T.round	ft.	in.
147	17	172
29	26	291

Dry Meaſure.

chal.	bu	pk.	gl.	qt.	pt.	in.
170	7	3	1	3	1	20
79	9	2	0	3	1	14

Wine Meaſure.

un.	*pipe.*	*hhd.*	*gln.*	*qt.*	*pt.*
47	0	1	45	2	1
19	1	0	60	3	0

Dry Meaſure.

butt.	*bb.*	*kil.*	*ſir.*	*gln.*	*qt.*	*pt.*
175	1	1	0	3	1	0
39	0	1	1	4	0	1

Time — **Motion.**

Cen. yr. ml. w. d. h. m. sec. Co. S ° ′ ″ ‴
n 18,11 5 2 3 18 30 10 ☉ will be in ♊ 1 2 17 34 37 10+ fr Aries 1810
n 18,10 5 3 6 18 30 5 — will be in ♊ 2 27 25 59 19+ from —
12 2 4 0 0 5 the Earth moves 11 20 8 37 51 } the Sun will be fr. Aries

This computation of time deviates from the Periodical Year 1.2564 d.

f A be in latitude 75° 35′ 20″ N. and in long. 7° 25′ 30″ West from W-
nd B in —— 42 23 0 N. —— 4 20 32 West from W.
hat is the dif-
erence of latitude & longitude ?

* * * * * *

REDUCTION

DESCENDING AND ASCENDING,

IS the changing of the form without altering the value.

RULE 1. To reduce higher denominations to lower; multiply the higheſt denomination given by ſo many of the next s make one of the higheſt; multiply this product by the ext denomination, &c. and add to the product of each deomination the given parts of the ſame name

2. To reduce lower denominations to higher; divide the iven denomination by ſo many as make one of the next igher; divide this quotient by the next higher denominaion, &c. if there by a remainder it is always of the ſame ame of the number divided.

EXAMPLES.

n 6 £ 12 *s* 6 *d.* ſterling, how many farthings ?

6 £. 12 *s.* 6 *d.*
20 s in 1 £.
132 s.=12+120 s.
12 d. in 1 ſhilling.
1584 d.
6 d. given +
590 x 4=6360 far. A.

In 6360 f. how many £ *s.* &c. ?

4)6360 f.
12)1590 d.
2|0)13|2 s. —r 6 d.
A 6 £. —r 12 s. 6d.

In 33 lb. 10 oz. 12 pwt. how many grains ?
33
12 + 10
406
20 + 12
8132
24
A. 195168 gr.

Reduce 195168 gr. to lb. &c.
24)195168(8132 pwt. ÷ 20
12)406 —r 12 pwt.
A. lb 33 --r 10 oz 12

In 13 cwt. 3 qrs. 12 lb. how many ounces ?
13 3 12 lb.
4
55 qr.
1552 lb.

In 24832 oz. how many cwt ?
A. 13 cwt. 3 qr. 12 lb.

In 58 pieces of holland, each containing 35 ells flemiſh, how many yds ?
58 x 35 = 2030 ells x 3 = 6090 qr. = 1522½ yd. A.

How many ft are there in 406944‴, ratio being in duodecimals ?
12) 12)
12)406944‴(33912″ = 2826′ in. = 235½ ft A.

How many inches, ſeconds & thirds are there in 235 ft 6 in.?
235 x 12 + 6 = 2826 in. = 33912″ = 406944‴ A.

How many times will a wheel 15 ft in circumference turn round in running 32 miles ?
11264 times. A.

In 40 acres how many rods ?
6400 rods. A.

In 17 gallons Wine, how many pints ? 136 pts. A.

Reduce 360° to ſeconds.
1296000″ A.

In 72 hhds. beer how many barrels ? 108 bar. A.

In 4527432 ſeconds how many weeks, &c. ? In the A. 7 w. 3 d. 9 h. 37 mn. 12 sec. how many minutes ?
75457 m. A.

Reduce ,743 £. to farthings
20
s. 14,860
12
d. 178,32
4
s. d. f.
4)713,28 f. A = 14 10 1,28
12)178,32 d.
20)14,86 s. (,743 £. pf.

educe 119 ſhillings 11 pence 3 farthings to £.

4)3,75,+11=11,75, ÷12=,97925,+119=119,97925

÷20=5,9989625 £. A.

20

s. 119,9792500

12

d. 11,75100 proof nearly.

4

f. 3,004

The miſtake is in ÷g by 12 if this be corrected the proof will be 119 s. 11d f. 2,9999999999999999, &c.

COMPOUND MULTIPLICATION,

IS the multiplying of a *ſimple* and a *compound* numer ; or two *compound* numbers together.

To multiply a ſimple and a compound number together.

Rule. Multiply the ſimple number by the firſt enomination of the compound ; divide this prouct by as many of the firſt denomination as make ne of the ſecond, ſet down the remainder, and add he quotient to the product of the ſecond denomiation, which find as before ; divide this product by s many as make one of the next denomination, ſet own the remainder and proceed as before to the laſt enomination, under which place its whole product.

Examples.

What will 9 barrels of flour come to at 4 £ 6 *s.* $11\frac{3}{4}$*d.* per arrel ?

	£.	*s.*	*d.*	*f.*
	4	6	11	3
				9
	36	54	99	4)27(6 *d*
qts.	3	8	6	24
		20)62(3 £	12)105(8 s	
		60	96	
	39	2	9	3 A.

What would 2375 quintals of fiſh come to at 3£. 10 *s*. 6 *d*. ſterling per quintal ?

2375
£3 10 6*d*
12)14250 *d*
1187 —r 6 *d*
23750 *s*
2|0)2493|7 *s*
1246 —*r* 17
7125

A. 8371 £. 17½ *s*.

	£.	*s*.	*d*.	*qui*.
or	3	10	6 x 5	
			10	
	35	5	0 x 7	
			10	
	352	10	0 x 3	
			10	
	3525	0	0 x 2	
			2	
	7050	0	0	
	1057	10	0	
	246	15	0	
	17	12	6	
	8371	17	6	A.

What is the value of 14 lb 10 oz 7 pwt 20 gr Troy at 2 *s*. per lb ?

14 10 7 20(+ 14 10 7 20=A
2 or proof.)

	£.	*s*	*d*.	*f*.
29*s* 8*d* $\frac{15}{20}$ $\frac{16}{24}$ A=	1	9	8	3$\frac{2}{3}$

What is the value of 5 yds 3 qr 2 na of cloth, at 1125 cts per yd ?

yards.	*qr*	*na*.
5	3	2
		1125
5625	3375	4)2250
984	362	
		562 —r2
6609	4)3937	or 562,5
	984 —r 1	
6609 cts.	1 qr.	2 A.

or 6609 $\frac{1\frac{1}{2}}{4}$ cts=66,09375 $.

What is the groſs weight and tare of 16 hhds of tobacco weighing, groſs 5 cwt 1 qr. 19 lb per hhd, tare 3 qr. 16 lb per hhd ?

cwt·*qr*. *lb*.
5 1 19
16

86 2 24 groſs } A.
3 16x16=14 1 4 tare }

5 1 19 x 8=43 1 12 } pf.
5 1 19 x 8=43 1 12 }

What would 114 acres 3 roods & 30 rods of land coſt at 2251 cts per acre ?

cts.
A. 258724$\frac{1\frac{1}{4}}{4}$ or 258724,3125

If the circumference of a wheel be 16 ft 6½ inches, how far will it move in turning round 40 times ?

ft 16 6' 6'' x 40=661 ft 8 in

If it be noon 2 hours 30 minutes 20 ſeconds ſooner at A han at B, what is their difference of longitude; the earth noving 15° each hour?

(*Table* 13)

2 h 30′ 20″
15°
37° 35′ 0 A.

When the simple number is such, that two or more numbers being nultiplied together will produce it, or nearly; the common method is, nultiply the compound by one of those simple numbers, this product by nother, &c. add to the last product, (or subtract from it.) the value of s many as the simple numbers are too small, (or too large.)

Or find the value of the whole at 1 far. d. s. pound, &c. per cwt. ound, yd,&c multiply this sum by the number of far. d. &c. in the rice, weight or measure and the product is the answer.

What is the value of 146 barrels f beef at 4 £. 2 s 6 d. per bar?

.46=12 x 12+2

49 10 0
12
594 0 0 price of 144
8 5 0 price of 2
A 602 5 0 146

What is the weight of 180 hhds of tobacco, each weighing 5 cwt. 2 qr 10 lb?

5 2 10
5 x 6 x 6=
27 3 22
6
167 2 20
6
1006 0 8 A

5cwt.2qr.10 lb.
180
28)1800
64—r 8
360
4)424
106—r 0
900 lb
A cwt 1006 0 8

What is the price of 129 lb at 9 d 3 f. per lb?

12) d. f. s. d. f.
4)129 = 32 —r 1= 2 8 1 price at 1 f. pr. lb.
9d. 3f. = 39 f = 6 x 6 + 3.
16 1 2
6
price at 36 f. £ 4 16 9 0
do. 3 8 0 3
price at 9¾ 5 4 9 3 A.
or 129 x 39 = 5031 f= 5£ 4s 9d 3f A.

What is the value at 120 \$ per cwt of 210 chests of tea each weighing 2 qr. 16 lb. neat?

cwt. qr. lb.
260 lb= 2 1 8 whole wt at 1 lb per chest.
2qr16lb= 72 or 6 x 12
167 0 16 whole weight.
120 \$ = 12 x 10
\$ 20057 2/4 16/28 A.

or 2 qr. 16 lb. = $\frac{72\text{ lb}}{1}$ x 260 x $\frac{120}{112}$ \$ = 20057,1428 + \$ A.

To multiply one compound number by another, increaſing in the ſame ratio, as Duodecimals.

Rule. Place the correſponding denominations under each other; multiply each denomination in the multiplicand by every denomination in the multiplier; divide and add the quotient as before, and ſet down each remainder ſo far to the right of its part of the multiplicand as its multiplier is to the right of the higheſt denomination given; then add the ſeveral products together, for the anſwer.

Parts multiplied by integers produce parts of integers, parts x*d* by parts produce parts of parts, &c. as, inches length x*d* by inches breadth produce ſquare inches, or 144ths of a foot; ſquare in. x*d* by in. depth produce cubic in. or 1728ths of a ft. in the ſame manner as tenths x*d* by tenths produce 100ths, or 100ths x*d* by 10ths produce 1000ths, &c The index of the x*r* added to that of the x*nd* is equal to the index of the pt.

EXAMPLES.

What is the contents of a board 12 ft. 6 in. long and 2 ft. 4 in. broad?

$$\begin{array}{lrrr} & 12\text{ ft} & 6' & \\ & 2 & 4 & \\ \hline 12\ 6'\times 2 = & 25 & 0 & '' \\ \text{—}\times 4' = & 4 & 2 & 0 \\ \hline \text{ft} & 29 & 2 & 0\ \text{A.} \end{array}$$

$$\begin{array}{lll} xy\ 2\frac{1}{2}\text{ ft by} & 2\frac{1}{2}\text{ ft.} & \text{ft}\ ' \\ 2\ 6' & 2\frac{1}{2} & 2,5 \\ 2\ 6 & 2\frac{1}{2}\text{ or} & 2,5 \\ \hline 5\ 0 & 5 & 1,2,5'' \\ 1\ 3\ 0'' & 1\frac{1}{4} & 5,0 \\ \hline A\ 6\ 3\ 0 = & 6\frac{1}{4} = & 6,2,5 \end{array}$$

The pt. is $6\frac{1}{4}$ ft=6 ft 36 sqr. in. or 6 ft. 2 tenths and 5 hundredths.

What is the ſolidity of a ſtick of timber $40\frac{1}{2}$ ft long, 1 ft. 2 in. broad, $11\frac{1}{2}$ in. thick?

$$\begin{array}{lllll} ft. & ' & '' & ''' & \\ 1 & 2 & & & \text{breadth} \\ & 11 & 6 & & \text{thickneſs} \\ \hline 1 & 0 & 10'' = 11' \times 1\ 2' & & \\ & & 7 & 0''' = 6'' \times 1\ 2' & \\ \hline 1 & 1 & 5 & & \text{area of end.} \\ 40 & 6 & & & \text{length.} \\ \hline 44 & 8 & 8'' = 40^{\circ} \times 1\ 1\ 5'' & & \\ & 6 & 8 & 6''' = 6' \times 1\ 1\ 5'' & \\ \hline 45 & 3 & 4 & 6\ \text{A.} = 45\frac{46}{144}\ 6 & \end{array}$$

How many cubic ft. in a bale of cotton 7 ft. 6 in long, 3 ft. 3 in. by 1 ft. 10 in.?

44 ft. 8′ 3″ A.

What is the contents of a load of wood, 6 ft. 4 in. long, 4 ft. broad and 3 ft. 10 in. deep?

ft 6 4 in.
4
25 4 area.= ft. ſq. in. 25 48
3 10
75 12 ″
0 250 40
A. 97 1 4 ſol.= ft. cub. in. 97 192

How many ſquare yds carpet will cover a floor 14 ft by 16?

yds. ft.
16 ft=5 1 ratio 3 16
14 = 4 2 14
20 4 9)224
0 10 2″ 24–r 8
A 24 $\frac{2}{3}$ $\frac{2}{3}$=24$\frac{8}{9}$ yds

What is the contents of a piece of cloth 20 yd 2 qr. 3 na. long, and 2$\frac{1}{2}$ yds. broad?

yd qr. na.
20 2 3 ratio 4.
2 2
40 4 6
40 4 6
yds 51 2 3 2 A.= sq.yd. qr. na. 51 11 8

What will 6$\frac{1}{2}$ ft of wood coſt at 2 s. 10 d. per ſt? 6° 6′
6 6 or 2 10
17 0 5° 5′ 0″
1 5 0 13 0
18 5 d 18 5 d A

One £ N. E. currency being worth 1 £. 5 s. N. J. what is 6 £ 10 s N. E. worth?

1 5
6 10
1 12 10″ } A.=8 £. 2 s 6d.

What will a triangular piece of land, 41 poles 4$\frac{1}{2}$ yds long, whoſe perpendicular breadth is 21 poles $\frac{1}{2}$ yd, coſt, at 10 $ per ſq. pole? 11 half yd=1 pl.

p. half yds.
41 9
10 6=$\frac{1}{2}$ perpendic.
418 2
22 8 10
440 10 10 contents.
1 eagle=10 $
A. E. 440 $\frac{10}{11}$ $\frac{10}{11}$=
4409,917 $

If two places differ in longitude 37° 35′, what is their difference of time, or how long will the earth be in moving 37° 35′ diurnal motion? The earth moves 1° in 4 mn. or 15° in an hour. [T. 13.]

37° 35′
0 4
h. 2 30 20″ A. =2$\frac{1}{2}$h. $\frac{1}{3}$m.

* * * * *

Compound numbers cannot be multiplied together, with ease or propriety, except they increase in the same ratio, or be reduced to the same denomination, (a simpl number.)

What is the ſquare and cube of 6 feet, 6 inches?
42 ft 3′ ſquare, 274 ft 7′ 6″ cube. A.

COMPOUND DIVISION,

IS the dividing of a *compound* number by a *ſimple*; or one *compound* number by another.

To divide a compound by a ſimple number.

Rule. Divide the higheſt denomination given by the diviſor; and if there be a remainder reduce it to the next lower denomination and add in the given parts of the ſame name, divide this ſum as before, &c. the ſeveral quotients will be the anſwer or quotient required.

Examples.

Divide 425 £ 15*s.* 10*d.* equally between 3 perſons.

£ s. d. f.
3)425 15 10 (141 18 7 $1\frac{1}{3}$ A.
3
141 — r 2£
20 + 15s 425 15 10 0 pf.
3)55
18 — r 1 s.
12 + 10 d.
3)22
7 — r 1
4
3)4
1 — r $\frac{1}{3}$

Divide 22 lb. 2 oz. 11 pwt. into 6 equal parts.

lb. oz. pwt. gr.
6)22 2 11 0
3 8 8 12 A. x 6 = pf.

If 25 hhds of ſugar weigh 276 cwt. 3 qr. 14 lb. what is the equal weight of each?

cwt qr lb oz dr.
25)276 3 14 (11 0 8 6 $6\frac{10}{25}$ A.
25
26
25
— r 1 x 4 + 3 = 7qr ÷ 25 = $0\frac{7}{25}$ qr.
28 + 14 lb.
25)210 (8 lb.
200
— r 10
16
25)160 (6 oz.
150 — r 10

If a board 20 ft long contain 40 ft $8\frac{1}{2}$′; what is its breath?

ft. in. ″
20)40 8 6 (2 0 $5\frac{2}{20}$. A.

If 4 pipes of Holland gin coſt 201 £ 12 ſ. ſterling; what did it coſt per gallon?

glns. £ *s.*
4 p. x 2 x 63=504)201 12(0 8 A.

When it is noon at Boſton, what time in the day is it at London, their difference of longitude being about 70° 58′?

15°)70° 58′(4 h. 43 mn. 52 ſec. after noon A.
or 4 o'clock and 44 minutes, nearly.

To divide by fractional parts; multiply by the lower and divide by the upper part of the fraction.

Divide 3 cwt. 2 qr. by $\frac{3}{4}$.
4
cwt. qr.
3)14 0(4 $2\frac{2}{3}$ A.

proof, [what is the weight of $\frac{3}{4}$ of 4 cwt. $2\frac{2}{3}$ qr.?]
3
4)14 0(3 cwt. 2 qr. A.

How many times is $\frac{3}{5}$ lb. contained in 4 lb. 6 oz.?
5
times.
3)21 14(7 $\frac{3\frac{2}{3}}{16}$ A.

To divide one compound number by another, increaſing in the ſame ratio.

Rule. Multiply the whole diviſor by the firſt figure or part of the quotient; ſubtract each part of the product from the ſame part of the dividend; in the ſame manner multiply the whole diviſor by the ſecond figure or part of the quotient, always obſerving to place the product or remainder of each denomination as far to the right (or left) of its part of the diviſor, as its part of the quotient is to the right (or left) of the higheſt denomination in the diviſor.

The names, of the ſeveral parts of the diviſor and quotient with the ſign x or *of* between them, repreſent every part of the dividend; as, (inches x*d* by inches i.e.) 12ths of 12ths repreſent 144ths, 144ths of 12ths=1728ths, &c. Alſo, the ſums of the arithmetical indices of the ÷*r* and *qt* repreſent the ſeveral parts of the dividend, and determine the place which each part of the work muſt occupy.

EXAMPLES.

The 1st (2d,3d,&c.) place of the qt— must occupy the same place in integers or parts which that figure or place . . . . . . of the ÷nd does, that stands over the units (2d, 3d, &c.) place . . of the ÷r in the 1st pt of the work.

$1^\circ 1' 2'')3^\circ 6' 9''(3^\circ 2' 11'' 6''' 6^{4}$

$3\ 3\ 6'' \equiv 1^\circ 1' 2'' \overset{+}{\times} 3^\circ$

3 3 0

$112'' \overset{+}{\times} 2' \equiv 2\ 2\ 4'''$

1 0 8 0

$112'' \overset{+}{\times} 11'' \equiv 1\ 0\ 0\ 10^{4}$

7 2 0

$112'' \overset{+}{\times} 6^{3} \equiv 6\ 7\ 0^{5}$

7 0 —r

What is the length of a board 2 feet 3½ in. broad containing 28 ft 11½′ ?

$\underbrace{2^\circ\ 3'\ 4''}_{\parallel})\underbrace{28^\circ 11'\ 6''\ 0'''}_{\parallel}(\underbrace{12^\circ\ 8'}_{\parallel} 6\tfrac{20}{41}''$ A.

or $328'')\ 4170''(12^{ft}$

3936

234

12

$328'')2808(8^{in}\ 6\tfrac{240}{328}''$ A.

What is the length of a ſtick of timber, containing 45 ft. 3′ 4″ 6‴, whoſe mean breadth is 1 ft. 2 inches, 11½ inches thick ?

$1^{ft}\ 2' \times 11^{in}\ 6'' = 1^{ft}\ 1'\ 5''$

$1^\circ\ 1'\ 5'')45^{ft}\ 3'\ 4''\ 6'''(40^{ft}\ 6^{in}$ A.

45 3 4 6

What muſt be the length of a floor 4 yd. 1 qr. 1 na. broad, to contain 23 yd. 3′ 3″ 3‴ 1⁗ ? *5yd. 2qr.* 1*na.* A.

What is the thickneſs of a grindſtone, containing 3ft 6′; whoſe area is 6 ft 10′?

$6^{ft}\ 10')3^{ft}\ 6'\ 6''(0^{ft}\ 6^{in}\ 2''\ 7'''+$A

What is the ſquare root of 42 ft. 3′ ?

ſuppoſe 6)42 3(7

42 6+

2)13(6ft.6in. A.

What is the cube root of 274ft. 7′ 6″ ? 7 0′

7x7=49)274 7′ 6″(5 6

3)1 6(6′

ft.7 −6′=6 ft. 6 in. A:

Eight £ 2ſ 6*d* N.J. currency being worth 6£ 10ſ N.E. what is 1£ N.E. worth ?

6£10′ ,8£2′10″(1£5ſ.N.J. A.

REDUCTION OF FRACTIONS,

IS the changing or bringing them from one poſition to anther, retaining the ſame value.

RULE 1. *To reduce fractions to their higheſt terms** ; Diide both the dividend and diviſor by ſuch a number as will o it without a remainder ; in the ſame manner divide their uotients, &c. the leaſt quotients poſſible expreſs the given alue in its ſhorteſt or higheſt terms. Revert the rule, to educe to lower terms.

EXAMPLES.

Reduce $\frac{4400}{8800}$ to its higheſt terms.

$$4)\frac{44|00}{88|00}\overset{11)}{=}\frac{11}{22}=\frac{1}{2}\text{ A.}\quad \frac{1}{2}\times\frac{44}{44}=\frac{44}{88}\text{ proof.}$$

Reduce $\frac{144}{240}$ £. to its higheſt terms.

$$4)\frac{144}{240}\overset{4)}{=}\frac{36}{60}\overset{3)}{=}\frac{9}{15}=\frac{3}{5}\text{ £. A.}\quad \times 4\times 3\times 4=\frac{144}{240}\text{ proof.}$$

Reduce $\frac{48}{56}$ to its higheſt terms. $8)\frac{48}{56}=\frac{6}{7}$ A.

Reduce $\frac{75}{25}$ to its higheſt terms. $25)\frac{75}{25}=\frac{3}{1}$ A.

Reduce $\frac{72}{94}$, $\frac{170}{84}$, $\frac{60}{125}$, $\frac{196}{182}$, $\frac{468}{1184}$, and $\frac{285}{114}$ to their higheſt terms. $\frac{36}{47}$, $\frac{85}{42}$, $\frac{12}{25}$, $\frac{14}{13}$, $\frac{117}{296}$, $\frac{5}{2}$, As.

RULE 2. *To reduce mixed or whole numbers to irregular fractions.* Multiply the integer by the diviſor of the fraction, and add in the dividend, place this ſum over the diviſor for the fraction required. Place 1 under any whole number and it becomes a fraction. Or multiply by any number and the product is the dividend and the multiplier the diviſor. Revert the rule, to reduce fractions to integers.

EXAMPLES.

Reduce $7\frac{1}{2}$ lb to a ſingle fraction.

$$7\times 2+1=\frac{15}{2}\text{ lb A.}\quad 2)15(7\tfrac{1}{2}\text{ proof.}$$

* * * * * *

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* If the reduction be ascending, (as that, by division, is called) the result must be *in higher terms* instead of lower ; if each one in the ÷nd *be 1 far. and be* reduced to 1 d. each i.e. be ÷d by 4, the reduction is *ſcending*,

Reduce $14\frac{15}{20}$ to a ſingle fraction.
$14 \times 20 + 15 = \frac{295}{20}$ £. A. = 195 s. ÷ 20 = $14\frac{3}{4}$ £ proof.

Reduce 5 galn. to a fraction having one, 2, 3, or 4 for its diviſor. $\frac{5}{1}$, $5 \times 2 = \frac{10}{2}$, $5 \times 3 = \frac{15}{3}$, $5 \times 4 = \frac{20}{4}$ gln. A. $5 = \frac{10}{2} = \frac{15}{3} = \frac{20}{4} = 20$ quarts.

Reduce $\frac{20\frac{1}{2}}{3}$ to a ſingle fraction. $\frac{20}{3} \times \frac{2}{2} + 1 = \frac{41}{6}$ A.

Reduce $\frac{20\frac{1}{2}}{3\frac{1}{2}}$ $\frac{20}{3} \times \frac{2}{2} + \frac{1}{1} = \frac{41}{7}$ A. $\frac{2}{2})\frac{41}{7}(\frac{20,5}{3,5}$

Reduce $\frac{1}{2}$ to a fraction whoſe ÷r is 4.
$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$ A.

Reduce $\frac{4}{2}$ to a fraction having 2 for its ÷nd.
$2)\frac{4}{2} = \frac{2}{1}$ A. or $\frac{4}{2} \times ,5 = \frac{2}{1,0}$.

Reduce $19\frac{2}{3}$, $12\frac{15}{17}$, $5\frac{3}{8}$, 20, 30, $\frac{354}{18}$, $\frac{2\frac{1}{2}}{1}$, $\frac{3}{2\frac{1}{2}}$ & $\frac{2\frac{2}{3}}{4\frac{2}{3}}$ to fractions &
‖ ‖ ‖ ‖ ‖ ‖ ‖ ‖ ‖ integers.
$\frac{59}{3}$, $\frac{219}{17}$, $\frac{43}{8}$, $\frac{20}{1}$, $\frac{60}{2}$, $19\frac{12}{18}$, $\frac{5}{2}$, $\frac{6}{5}$ & $\frac{14}{14}$ As.

RULE 3. *To reduce compound to ſimple fractions*; Multiply all the dividends together for the dividend, and all the diviſors for the diviſor of the fraction required. It is unneceſſary to multiply by ſuch numbers as are alike above and below the line. Revert the rule, to reduce ſimple to compound fractions. This rule anſwers for x*n*.

EXAMPLES.

Reduce $\frac{3}{4}$ of $\frac{4}{3}$ of $\frac{5}{7}$ of $\frac{4}{9}$ to a ſimple fraction.
$\frac{3}{4} \times \frac{4}{3} \times \frac{5}{7} \times \frac{4}{9} = \frac{24|0}{126|0}$ A. $= \frac{4}{21}$. or $\frac{3}{\underset{4}{1}} \times \frac{\overset{4}{1}}{\underset{5}{1}} \times \frac{\overset{5}{1}}{7} \times \frac{4}{9} = \frac{12}{63}$
$\frac{3}{4})\frac{240}{1260}(\frac{80}{315} \div \frac{4}{3} = \frac{20}{63} \div \frac{5}{7} = \frac{4}{9}$ $\div \frac{4}{9} = \frac{1}{1}$ proof.

Reduce $\frac{1}{2}$ of $\frac{1}{10}$ \$ to a ſingle fraction.
$\frac{1}{2} \times \frac{1}{10} = \frac{1}{20}$ \$ A. $= \frac{1}{2}$ di. $\frac{1}{2})\frac{1}{20} = \frac{1}{10}$ proof.

Reduce $\frac{1}{2}$ *of* $\frac{4}{5}$ *of* $\frac{8}{2}$ bu. $\frac{1}{2} \times \frac{4}{5} \times \frac{8}{2} = \frac{32}{20}$ bu. A.

Reduce $\frac{3}{4}$ *of* a pint to gln. $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{1}{4} = \frac{3}{32}$ gln. A.

Reduce $\frac{3}{32}$ of a gln to pt. $\frac{1}{8})\frac{3}{32}=\frac{3}{4}\div\frac{1}{1}=\frac{3}{4}$ pt. A.
Or $\frac{3}{32}$ of $\frac{4}{1}$ of $\frac{2}{1}=\frac{24}{32}$ pt. A. $=\frac{3}{4}$ pt.

Reduce $\frac{1}{2}$ of $\frac{10}{1}$ cts, $\frac{1}{2}$ of $\frac{1}{12}$ lb, $\frac{3}{4}$ of $\frac{1}{4}$ d of $\frac{1}{12}$ s, $\frac{3}{4}$ of $\frac{1}{20}$ tun to ſimple fractions.
$\frac{10}{2}$ cts. $\frac{1}{24}$ lb. $\frac{3}{192}$ s. $\frac{3}{80}$ tun. As.

RULE 4. *To reduce one denomination, Coin, Weight, Meaſure, &c. to any other;* Multiply the denomination of the Coin, &c. given by the *proportion** it bears to that which is required, and the product is the anſwer: divide the dividend of any fraction by its diviſor, (multiply the remainder by the next denomination or proportion, and divide as before;) and the quotient is equivalent to the primitive fraction.

EXAMPLES

Reduce $\frac{30}{2}$ s. to £. and to \$. (pro. between s. and £ is $\frac{1}{20}$, and between s. and \$. $\frac{2}{9}$. *T.* 1. as 2 : 9 :: 1 s. : 1 \$.)
$\frac{30}{2}\times\frac{1}{20}=\frac{30}{40}$ £. A. 4)3,00(,75 £ $=\frac{3}{4}$ £. $\frac{30}{2}\times\frac{2}{9}=\frac{60}{18}$ \$ A.
8)60,00(3,333$\frac{1}{3}=\frac{60}{18}$

Reduce 2 lb. 3 oz. Troy to Avordupois (pro. $\frac{576}{700}$ *T.* 2.)
$2\frac{3}{12}=\frac{27}{12}$, x $\frac{576}{700}=\frac{15552}{8400}$ A.
84|00)155|52(1 lb. 13 oz. $9\frac{8112}{8400}$dr. $=\frac{15552}{8400}$ lb. Avor.

In 6 cubic ft how many buſhels?
(pro. $\frac{298}{371}$ *T.* 9.) $\frac{6}{1}$ x $\frac{298}{371}=\frac{1788}{371}$ bu. A. = 4 bu. $3\frac{103}{371}$ pk.

Reduce $\frac{3}{2}$ bu. to wine gallons.
(pro. $\frac{624}{67}$ *T.* 10) $\frac{3}{2}$ x $\frac{624}{67}=\frac{1872}{134}$ gln. A. $=13\frac{130}{134}$

What is the weight of 3 wine glns of water? (pro. $\frac{167}{20}$ *T.* 11)
3 x $\frac{167}{20}=\frac{501}{20}$ lb. Avor. A. $=25\frac{1}{20}$ lb.

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* The proportion, which one denomination, Coin, &c. bears to another, may be found in the tables of the same name. In all cases if the denomination, &c. given be larger than that which is required the ÷nd of the fraction or proportion must be larger than the ÷r, if less the reverse.

In 231 ale and beer glns, how many wine glns ?
231 x 282 ÷ 231 = 282 wine gln. A.

In how many days does the earth move 1' ſigns ?
$1\frac{1}{2}=\frac{3}{2}\times30{,}438=45{,}657$ days. A.

How far does the earth move in its orbit in $\frac{2}{3}$ of an hour ?
$\frac{2}{3}\times68215{,}67=45477{,}113$† miles. A.
or 98,56+ ſeconds A. = (10° Diurnal motion.)

Reduce 3 £. N E. & Vir. currency to N. J. &c. to N. Y. and to ſterling.
(pro $\frac{5}{4}$, $\frac{4}{3}$, $\frac{3}{4}$ T. 15) $3\times\frac{5}{4}=\frac{15}{4}$ £. N. J. = 3£ 15 s.
$3\times\frac{4}{3}=\frac{12}{3}$ N Y. $3\times\frac{3}{4}=\frac{9}{4}$ ſter.
3 £ 15 s N. J. = 4 £ N. Y. = 2 £ 5 s ſter. As.

Reduce 10£ 3 s N. J. to \$, to N. E. and to ſterling.
10£. 3 s. = 203 s $\times\frac{2}{15}=\frac{406}{15}=27\frac{1}{15}$ \$. A.
10 3 $\times\frac{4}{5}$ = 8£. 2,4 s. or $\frac{40}{5}$ £. $\frac{12}{5}$ s. N. E. A.
10 3 $\times\frac{3}{5}$ = 6£. 1 s. 9,6 d. ſter. A.

Rule 5. *To reduce fractions to thoſe having a common diviſor;* Multiply all the diviſors of the given fractions together for the common diviſor required ; multiply each dividend into all the diviſors except its own for the dividends required. Or multiply or divide any fraction by ſuch a number as will bring it to the ſame ÷r of thoſe given to the other fractions.

Examples.

Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{4}{5}$ to fractions having a common ÷r.

$$\left.\begin{array}{l}\frac{1\times3\times4\times5}{2\times3\times4\times5}=\frac{60}{120}\\ \frac{2}{3}\times\frac{2}{2}\times\frac{4}{4}\times\frac{5}{5}=\frac{80}{120}\\ \text{com. ÷r}\end{array}\right\}\text{As.}\left\{\begin{array}{l}\frac{90}{120}=\frac{3}{4}\,(\times\frac{2}{2}\times\frac{3}{3}\times\frac{5}{5}\\ \frac{96}{120}=\frac{4}{5}\,(\times\frac{2}{2}\times\frac{3}{3}\times\frac{4}{4}\end{array}\right.$$

Reduce $\frac{3}{4}$ of $\frac{1}{2}$, $\frac{3}{2}$ and 2 to fractions having a common diviſor.
$\frac{6}{16}$, $\frac{24}{16}$ and $\frac{32}{16}$ A.

$$\begin{array}{l}\frac{3}{4}\text{ of }\frac{1}{2}=\frac{3}{8}\\ 2=\frac{2}{1}\end{array}\quad\left.\begin{array}{l}3\times2\times1=6\\ 3\times8\times1=24\\ 2\times8\times2=32\end{array}\right\}\text{÷nds.}\left\{\begin{array}{l}\text{or }\frac{3}{8}\times2=\\ \frac{3}{2}\times2=\\ \frac{2}{1}\times4=\end{array}\right\}\text{A.}$$

÷rs 8 x 2 x 1 = 16 com. ÷r.

Reduce $\frac{1}{3}$, $\frac{3}{4}$ of $\frac{4}{5}$, $5\frac{1}{2}$ and $\frac{1}{3}$ to a common $\div$*r.*

$\frac{190}{570}$, $\frac{342}{570}$, $\frac{3135}{570}$, $\frac{190}{570}$ A.

ADDITION OF FRACTIONS.

Rule. Reduce all the fractions to those having a common divisor; add the dividends together, for the dividend of the fraction required; this sum divided by the common divisor gives a quotient equivalent to all the fractions of which it is formed.

Examples.

Add $1\frac{1}{2}$, $2\frac{3}{4}$, $11\frac{2}{3}$ and $\frac{1}{2}$ together.

$\div$*ds.* $\div$*rs.*

$1 \times 4 \times 3 \times 2 = 24$

$3 \times 2 \times 3 \times 2 = 36$ 11

$2 \times 2 \times 4 \times 2 = 32$ 2

$4 \times 2 \times 4 \times 3 = 96$ 1

$\div$*rs* $2 \times 4 \times 3 \times 2 = 48)188(3\frac{44}{48} + 14 = 17\frac{44}{48}$ A.

Add $21\frac{15}{20}$ £. $14\frac{10}{20}$, $7\frac{19}{20}$ together.

14 10 s.

7 19 s.

£. 44 4 s A.

Add $6\frac{25}{28}$, $3\frac{19}{28}$ and $\frac{12}{14}$ qr. together.

3 19 lb.

24 $\frac{12}{14} \times 2 = \frac{24}{28}$

A. $11\frac{12}{28}$

Add $37\frac{1727}{1728}$ ft. $\frac{1275}{1728}$ and 1640 cubic in. together.

A. 39 ft. 1186 cubic in.

Add $2\frac{1}{2}$, $\frac{1}{2}$ of 3 and $\frac{4}{8}$ together.

$2\frac{1}{2} = \frac{5}{2}$, $\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$, $4)\frac{4}{8} = \frac{1}{2}$. $4\frac{1}{2}$ A.

Add ſeven $\frac{\text{two}}{\text{thirds}}$ gallons, 3 $\frac{\text{one}}{\text{half}}$ to four $\frac{2}{6}$. Fifteen $\frac{\text{three}}{\text{sixths}}$ A

÷*nds.* *diviſors.*
two x two x ſix= 24 ſeven
one x three x ſix= 18 three
two x three x two= 12 four
÷*rs* three x two x ſix=36)54(1 $\frac{18}{36}$+ fourteen=fifteen $\frac{\text{one}}{\text{half}}$. A.

SUBTRACTION OF FRACTIONS.

Rule. Reduce the fractions to thoſe having a common diviſor, and the difference of the dividends placed over the common diviſor is the anſwer. When there are integers and fractions given proceed as in compound ſubtraction.

Examples.

Subtract $\frac{7}{8}$ from $\frac{15}{16}$.

$\frac{7}{8}$ x $\frac{2}{2}$=$\frac{14}{16}$ −$\frac{15}{16}$=$\frac{1}{16}$ A, or $\frac{8}{128}$

From 25$\frac{15}{20}$ £. take 17$\frac{18}{20}$ £.
17 18 s.

7 $\frac{17}{20}$ A.

From $\frac{1}{4}$ £. take $\frac{9}{10}$ s.

£. $\frac{1}{4}$ x $\frac{20}{1}$=$\frac{20}{4}$ s. x $\frac{10}{10}$=$\frac{200}{40}$ −$\frac{36}{40}$=$\frac{164}{40}$ s. A.

From $\frac{3}{4}$ oz. take $\frac{3}{4}$ pwt.

$\frac{3}{4}$ oz.=$\frac{3}{4}$ of $\frac{20}{1}$=$\frac{60}{4}$ pwt. $\frac{57}{4}$ pwt. A.=14$\frac{1}{4}$

From 37$\frac{1}{4}$ take 19$\frac{4}{7}$. 17$\frac{19}{28}$ A.

From 13$\frac{1}{3}$ take $\frac{3}{4}$ of 15. 2$\frac{1}{12}$ A.

Subtract two $\frac{\text{three}}{\text{fourths}}$ from four $\frac{\text{one}}{\text{half}}$. one $\frac{\text{three}}{\text{fourths}}$ A.

MULTIPLICATION OF FRACTIONS.

Rule. Reduce the numbers to ſimple fractions, and multiply them all together, as by Rule 3, for the fraction requird ; or to *multiply integers by fractions* ; multiply by the diviend and divide by the diviſor of the fraction. Or proceed as a compound x*n*.

EXAMPLES.

Multiply $4\frac{1}{2}$ by $\frac{2}{5}$ $4\frac{1}{2}, = \frac{9}{2}, \times \frac{2}{5} = \frac{18}{10}$ A.

Multiply $2\frac{1}{2}$ by $2\frac{1}{2}$. $\frac{5}{2} \times \frac{5}{2} = \frac{25}{4}$ A.

or $2\frac{1}{2} \times 2 = 5, + 1\frac{1}{4} (= 2\frac{1}{2} \times \frac{1}{2}) = 6\frac{1}{4}$ A.

What is the ſolidity of a block 2 ft. long $1\frac{3}{4}$ broad and $1\frac{1}{2}$ hick? $2 \times \frac{3}{2} = 3, \times \frac{7}{4} = 5\frac{1}{4}$ ſt. A.

What is $\frac{1}{4}$ of $7\frac{1}{4}$. $7\frac{1}{4} \times \frac{1}{4} = 1\frac{13}{15}$ A.

xy $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$. $\frac{5}{14}$ = A.

What is the diſcount on 400 $ at $\frac{6}{106}$?

$400 \times \frac{6}{106} = \frac{2400}{106}$ A.

What is the contents of a piece of land $\frac{3}{4}$ of a mile long and $\frac{1}{2}$ broad? $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ mi. A. = 240 acres.

$\frac{3}{4}$ mi. = 240 rods, $\frac{1}{2}$ mi. = 160 rods, $\times \frac{240}{160} = 240$ acres.

What is the contents of a board $6\frac{3}{12}$ ft. long $2\frac{7}{12}$ ft. broad?

$\frac{75}{12} \times \frac{31}{12} = \frac{2325}{144}$ ft A. = 6 ft. 3′ x 2 ft. 7′ = 16 ft. 1′ 9″.

DIVISION OF FRACTIONS.

RULE. Proceed as in multiplication, after inverting the fraction to divide by ; or divide the upper and lower part of the dividend by thoſe of the diviſor. Or *to divide integers by fractions* ; multiply by the diviſor and divide by the dividend of the fraction. *To divide fractions by integers* ; multiply the diviſor of the fraction by the integer, or divide the dividend by it. *To divide by integers and fractions, or mixt numbers* ; reduce the fraction to decimals, or reduce the ÷*r* and ÷*nd* to the ſame denomination (i.e. xy both by the ÷r *of the fraction) and* divide as before.

Each of the above methods may be varied to the firſt.

EXAMPLES.

Divide $\frac{3}{4}$ by $\frac{1}{4}$. ÷ inverted $\frac{4}{1} \times \frac{3}{4} = \frac{12}{4}$ or $\frac{1}{4})\frac{3}{4} = \frac{3}{1}$ A.=3.

Divide $\frac{1}{4}$ by $\frac{3}{4}$. ÷r inverted $\frac{4}{3} \times \frac{1}{4} = \frac{4}{12}$ A.=$\frac{1}{3}$

Divide 4 by $\frac{1}{4}$. $4 \times \frac{4}{1} = \frac{16}{1}$ A.= 16

Divide $\frac{1}{4}$ by 4. $\frac{1}{4} \times {}_4 = \frac{1}{16}$ A.

Divide 7 by 9. $\frac{7}{1} \times \frac{1}{9} = \frac{7}{9}$ A.

Divide 76 by $4\frac{1}{2}$.

$4\frac{1}{2} = 4,5)76(16,88+$ or $4\frac{1}{2})76$

45   2   2

31,0   9)152(16,88+ A.=$16\frac{8}{9}$

Divide $42\frac{3}{4}$ by $2\frac{1}{2}$. $2\frac{1}{2})42\frac{3}{4}$

4   4

or $5)85\frac{1}{2}(17,1$ A.   10)171(17,1 A. 2,5)42,75(17,1 A

If $2\frac{1}{2}$ lb. coſt $\frac{12}{3}$ of 2 cts. what will 1 lb. coſt ?

$2\frac{1}{4} = \frac{5}{2}$ ÷r. $\frac{12}{3} \times \frac{2}{1} = \frac{24}{3}, \times \frac{2}{5} = \frac{48}{15}$ A.=$3\frac{1}{5}$ cts.

What is the ſum whoſe diſcount is 20 £. at $\frac{6}{106}$?

$\frac{20}{1} \times \frac{106}{6} = \frac{2120}{6}$ A.

What is the width of a board $8\frac{1}{2}$ ft.long containing $18\frac{3}{4}$ ft.?

$8\frac{1}{2} = \frac{17}{2}$ ÷r, $18\frac{3}{4} = \frac{75}{4} \times \frac{2}{17} = \frac{150}{68}$ A.=$2\frac{14}{68}$ ft.

Divide 76 by $16\frac{8}{9}$ $4\frac{1}{2}$. A.

How many times is $\frac{1}{8}$ contained in 4 ?

$\frac{4}{1} \times \frac{8}{1} = \frac{32}{1}$ A.=32 times.

Divide $7\frac{2}{3}$ by 2. $2)7\frac{2}{3}$

A. $3\frac{5}{6}$ or $3\frac{1}{2}$ and $\frac{1}{3}$ or $\frac{23}{6}$

How many times is $\frac{\text{ten}}{\text{fourths}}$ oz. contained in ſix lb. $\frac{16}{2}$ oz. *troy ?*

6 lb. $\times \frac{12}{1} = \frac{72}{1}$ oz. + $\frac{16}{2} = \frac{80}{1}, \times \frac{4}{10} = \frac{320}{10}$ A=32 times.

nd. xr.	pt.	qt.
x ÷8=	32	½
x ÷4=	16	1
x ÷2=	8	2
x ÷1=	4	
x ÷½=	8	2
x ÷¼=	16	1
x ÷⅛=	32	½
÷nd ÷r	qt.	pt.

The reſults from xn and ÷n are always directly opposit; the product of integers being larger than the qt, in the same proportion the qt of a regular fraction is larger than the pt; hence 1, is the only medium, by which we may both xy and ÷e without altering the number, and from which xn increases and decreases, and from which ÷n decreases and increases (the number) to the reverse.

THE RULE OF THREE,

IN COMPOUND NUMBERS,

IS an application of Simple Proportion, by which numbers, whoſe diviſion is not decimal, are compared.

RULE. Obſerve the ſame method of ſtating and uſing the numbers as in ſimple proportion, except they muſt be multiplied and divided as compound numbers; or, reduce the two given numbers that are alike, or of the ſame quality, to the ſame denomination, and the number that is like the anſwer, to a ſingle denomination, then multiply and divide as in ſimple numbers, and the quotient is the anſwer in the ſame denomination of that to which the ſecond term was reduced.

EXAMPLES.

If 9 cwt 3 qr. of ſugar coſt 27 £. 17 s. 6 d. what will 2 cwt 1 qr 11 lb coſt?

As 9 cwt 3 : 27 £ 17 6 :: 2 cwt 1 qr 11 lb : 6 £ 14 s 3 d A.

4	20	4
39	557	9
28	12	28
1092 lb :	6690 d ::	263 lb : 1611,23 d A=6 £ 14 s 3,23
	263	

1092)1759470(1611,23 + d.

as 1611,23 d : 263 lb :: 6690 d : 1092 lb. proof.

If a ſhip coſt 4752 £ 15 s what is ⅜ of her worth?

As 8 : 4752 £ 15 s :: 3 : 1782 £ 5 s 7 d 2 far.

If 4 yds coſt 2 £ 6 s 4 d; what will 60 yds coſt ?
As 4 yds : 2 £ 6 s 4 d :: 60 yd : 34 £ 15 s A.
60

4)139 0 0

£ 34 —r $\frac{3}{4}$ x 20=15 or 2 £ 6 4 x $^{1}\frac{5}{1}$=34 £ 15 A.
As 34 £ 15 : 2 £ 4 6 : 60 yds : 4 yd. proof.

If 22$\frac{1}{2}$ ft coſt 2 s 4 d; what will 223 ft 4′ of the ſame coſt?
As 22 ft 6′ : 2 s 4 d :: 223 ft 4′ : 23 s 1 d 10″ A=1 £ 3 1$\frac{10}{12}$ d
2 4

446 8
74 5 4″

22 6)521 1 4(20 s
22 6′ x 20= 450 0

71 1 (3 s
—— x 3= 67 6

3 7 4 (1′d
—— x 1′= 1 10 6

1 8 10 (10″
—— x 10″= 1 6 9

2 1 —r

s d. ft.
or 22$\frac{1}{2}$: 2 4 :: 223$\frac{4}{12}$
12 12 12
270 : 28d :: 2680 :
277,93 d. A. proof.

How many yards of cloth may be bought for 57 £ 13 s, whereof 9$\frac{1}{2}$ yds coſt 3 £ 15 s 5$\frac{1}{2}$ d ?
As 3 £ 15 s 5$\frac{1}{2}$ d : 9$\frac{1}{2}$ yds :: 57 13 : 145yds 0 qr 2,5 + na. A.
20 4 20

75 38 1153
12 12

905,5 d : 38 qrs :: 13836 d : A.
38

905,5)525768(580,638 qrs A=145,159 yd

Bought 9 cheſts of tea each weighing 3 cwt 2 qr 21 lb at 4£ 9 s per cwt ; what came they to ?
As 1 cwt : 4£ 9 s :: 3 cwt 2 qr 21 lb x 9 : 147 £ 13 s $8\frac{1}{4}$ d A

If 100£ in 12 months gain 5£ 10 s ; how much will 406£ 15 s gain in $9\frac{1}{2}$ months ?
As 100 x 12 : 5 £ 10 s :: 406 £ 15 s x $9\frac{1}{2}$: 17£ 14 2 2,15 A

If a perſon allows 2 £ 10 s per cent premium on 456 £ 15 s (foreign money) ; what is the whole ſum allowed ?
As 100 + 2£ 10 s : 2£ 10 s :: 456£ 15 s : 11£ 2 s 9 d 2,6 f A.

What is the diſcount of 635 £ 17 s due 2 year's hence, at $5\frac{1}{2}$ per cent per year ?

$5\frac{1}{2}$ x 2=11 per cent.

As 100+ 11 : 11 :: 635 £ 17 s : 63 £ 0 s $2\frac{3}{4}$+ A.

If a factor receive 455 £ 10 s to purchaſe goods after having deducted his premium of 3 £ 10 s per cent on the purchaſe ; what value of goods were purchaſed ?
As 100+3 10 : 100 :: 455 10 : 440 £ 1 10 $2\frac{178}{207}$ A.

What is the commiſſion on 1371 £ 9 s 5 d at 5 per cent ?
As 100 £ : 5 £ :: 1371 £ 9 5 : 68 £ 11 s $5\frac{1}{2}$ d A.

What is the brokerage on 1321 £ 11 s 4 d at $1\frac{1}{8}$ per cent ?
14 £ 17 s 4 d A.

" What is the premium of inſuring 924 £ at 7 per cent ?
As 100 : 7 :: 924 : 64 £ 13 s 7 d." A.

" A merchant ſent a veſſel and cargo to ſea, valued at 1525 £ ; what ſum muſt the policy be taken out for, to cover his property, premium $19\frac{1}{2}$ per cent ?"
As 100 −19,5 : 100 :: 1525 : 1894 £ 8 $2\frac{1}{4}$ A.

A and B buy certain merchandize, amounting to 120 £ of which A pays 80 £ and B 40, on which they gain 32 £ ; what ſhare of the gain belongs to each ?

$$\textit{As } 80+40 : 32 :: \left\{\begin{array}{l} 80 : 21\text{ £ } 6 \;\; 8 \text{ A's gain.} \\ 40 : 10\text{ £ } 13 \;\; 4 \text{ B's gain.} \end{array}\right\}$$

A and B trade in company, A puts into ſtock 950 £ for 5 months, B 785 for 6 months, and by trading they gain 275 £ 18 s 4 d ; what is the profit of each ?

$$\left.\begin{array}{l}\text{A's ſtock } 950\times5=4750\\ \text{B's ſtock } 785\times6=4710\end{array}\right\} : 275\ 18\ 4 :: \left\{\begin{array}{l}4750 : 138\ 10\ 10\ \text{A's}\\ 4710 : 137\ 7\ 6\ \text{B's.}\end{array}\right.$$

If 5 £ 10 s be gained by ſelling 43 yds of cloth ; what is the average gain per yd ?

As 43 yds : 5 £ 10 :: 1 yd : 0 £ 2 s 6 d 3 –f. A.

How many yards of cloth at 8 s 6 d per yd will pay for 250 gallons of brandy at 9 s 4 d per gallon ?

As 8 s 6 d : 250 gln :: 9 s 4 d : 274 yd 2+ qr A.

If 1 cwt 2 qr 14 lb reſt on the end of a lever 8 ft 6 in. from a prop ; what weight will it balance on the other end 1 ft 2 in. from the prop ?

As 1 ft 2 : 1 cwt 2 14 :: 8 ft 6 : 11 cwt 3 qr 10lb A.

If the end of the bar of a printing preſs move 3 ft 3 in. while the plattin or ſcrew deſcends one inch ; what weight or ſtrength muſt be applied to the end of the bar, that the preſſure may be equal to 1 tun 3 cwt, allowing nothing for friction ?

As 3 ft 3′ : 1 tun 3 :: 1 inch : 2 qr. $10\frac{2}{39}$ lb. A.

If the tunnage of a veſſel, whoſe length is 76 ft. breadth 24 ft and depth 11 ft 8 in. be 180 tuns ; what is the tunnage of a veſſel, whoſe form is ſimilar to the above, and the dimenſions taken in ſimilar places (being nearly proportional) are, 81 ft length, 25 ft 6 in. breadth, and 12 ft 4 in. depth ?

As 76x24x11 8 : 180 T :: 81x25 6x12 4 : $215\frac{1024}{2128}$ T A.

Common method of ſtating PROPORTION *inverſely or indirectly.*

RECIPROCAL OR INVERSE PROPORTION,

" *Teaches,* by having three numbers given, to find a fourth, *which ſhall* bear the ſame proportion to the ſecond, as the *firſt bears* to the third.

Therefore, the greater the third term is, in compariſon with the firſt, the leſs the fourth term will be compared with the ſecond; or, the leſs the third term is in proportion to the firſt, the greater the fourth muſt be in proportion to the ſecond; and this is called *reciprocal, inverted* or *indirect* Proportion."

"RULE. Make the number that is the demand of the queſtion, the third term, the number that is of the ſame name or quality, the firſt term, and the remaining number will be the middle term. Multiply the firſt and ſecond terms together, and divide the product by the third, and the quotient is the anſwer."

EXAMPLES.

"If 100 workmen complete a piece of work in 12 days, how many are ſufficient to do it in 3 days?"

As 12 d : 100 m :: 3 d : 400 men A. inverſely.
12
―――
3)1200(400 m.

Or as 1 piece of work : 12 d x 100 m :: 1 piece of work : 3 d x 400 men. A. directly.

"If 8 boarders drink a barrel of cider in 12 days; how long would it laſt if 4 more came among them?"

As 8 : 12 :: 12 : 8 days. A. inverſely.
Or as 1 barrel : 8 m x 12 d :: 1 bar. 12 m x 8 d A. directly.

"What length of board 7½ in. wide will make a ſquare foot?"

As 12 br : 12 lg :: 7½ br : 19⅕ lg. A. inverſely.
Or 1 ft : 12 x 12 :: 1 ft : 7½ x 19⅕ A. directly.
Or 1 ft=144 ſq. in. ÷7,5=19⅕ A.

"A wall which was to be built 24 ft high, was raiſed 8 ft by 6 men, in 12 days; how many men muſt be employed, to finiſh the wall in 4 days?"

As 8 : 6 :: $\overline{24-8}$: 12. Then as 12 d : 12 m :: 4 d : 36 m A.
Or as 8 ft : 12 d x 6 m :: $(\overline{24-8}=)$ 16 ft : 4 d x 36 men A

Common method of ſtating PROPORTION *when more than three terms are given.*

COMPOUND PROPORTION.

“ COMPOUND PROPORTION teaches to reſolve ſuch queſtions, as require two or more ſtatings by ſimple proportion.

“ RULE. State the queſtion, by placing the three conditional terms in this order: that which is the principal cauſe of gain, loſs, or action, poſſeſſes the firſt place; that which denotes ſpace of time or diſtance of place, the ſecond; and that which is the gain, loſs, or action the third; then place the other two terms, which move the queſtion, under thoſe of the ſame name, and if the blank place fall under the third, multiply the three laſt terms for a dividend, and the two firſt for a diviſor: but if the blank fall under the firſt or ſecond place multiply the firſt, ſecond, and laſt terms together for a dividend, and the other two for a diviſor; and the quotient will be the anſwer.”

“ If the proportion be direct, the ſixth term ſought muſt bear ſuch proportion to the fourth and fifth, as the third bears to the firſt and ſecond: But if inverſe, the ſixth number muſt bear ſuch proportion to the fourth and fifth, as the firſt bears to the ſecond and third.”

EXAMPLES.

“ If 100 £ in 12 months gain 5, how much will 400 gain in 3 months?

prin. time. int. prin. time. int.

100 £ : 12 m :: 5 £ Or as 100 x 12 : 5 :: 400 x 3 : 5 £ A.

400 : 3 100 1200 £ : 5 £ :: 1200 £ : 5 £

3 12

1200 x 5=600 ÷1200=5 A.

“ If 8 men ſpend 32 £ in 13 weeks; what will 24 men ſpend in 52 weeks?”

8 m : 13 w :: 32 £ m w £ m w £

24 : 52 384 A. Or as 8 x 13 : 32 :: 24 x 52 : 384 A.

“ *If the* freight of 9 hhds. of ſugar, each weighing 12 cwt. 20 *leagues* coſt 16 £; what muſt be paid for the freight of 50 *tierces* ditto, each weighing 2½ cwt. 100 leagues?”

```
9 hhd  :  20 l. :: 16 £.
o      :  100   A. 92 £ 11 10⅔.
: 108 cwt x 20 l : 16 £ :: 125 cwt x 100 l : 92 £ 11 10 2/9  A.
```

THE RULE OF THREE,

IN ARITHMETICAL PROPORTION.

s a compariſon of numbers whoſe difference is conſidered as
ie *ratio*, and is performed by addition and ſubtraction.

Rule. Place the antecedent for the firſt term, whoſe con-
quent is given, which place for the ſecond term ; and place
ie antecedent for the third term, whoſe conſequent is re-
iired, which will be the fourth term ; to find which add
ie ſecond and third terms together & ſubtract the firſt term
om their ſum ; or if the firſt term be leſs than the ſecond
ld their difference to the third for the fourth ; but if the
ſt be larger than the ſecond ſubtract their difference from
e third, and the remainder is the fourth term : in the ſame
anner proceed for the 5th, 6th, &c. term when required.

Examples.

If 2 be the antecedent, 5 its conſequent ; what is the con-
quent of 8 ? As 2 .. 5 :: 8 .. 11 A. arithmetically.

```
                              +5
                              —
s 2 .. 5 :: 8 .. 11 A.        13    Or as 2 .. 8 :: 5 .. 11 A.
  -2   3+                    - 2              2    6
  —    —                      —               —    —
  3   11                      11              6   11
```

If 18 yds coſt 24 $ what will 33 yds coſt ? 1,25527 log-
ithm or index of 18 ; 1,38021 lgr. of 24 ; 1,5185 lgr. of 33
As 18 yd : 24 $ :: 33 : 44 $ geometrically.
s 1,25527 .. 1,38021 :: 1,51851 . 1,64345 arithmetically.

```
          1,51851
         ————————   or 1,25527 .. 1,38021 :: 1,51851 ..
          2,89872              -1,25527   +,12494
          1,25527              ————————   ————————
         ————————                ,12494   1,64345
          1,64345 logarithm of 44 $ A.
```

What is the length of a fiſh, whoſe head is 9 inches, his tail $\frac{1}{3}$ the length of his head and $\frac{1}{8}$ the length of the whole fiſh, and his body as long as head and tail both ?

8)
Suppoſe his length 24
then his tail is { 3 = $\frac{1}{8}$ fiſh, 3 = $\frac{1}{3}$ head
head 9
body 15
30 − 24 = 6 + error

ſuppoſe his length 40
then his tail is { 5 = $\frac{1}{8}$ fiſh, 3 = $\frac{1}{3}$ head
head 9
body 17
34 − 40 = 6 − error, or diſagreement

ſup. error.
24 x 6 − = 144
40 x 6 + = 240
2)64   12)384
8)32 = $\frac{1}{2}$ ſup. A. 32
tail { 4 = $\frac{1}{8}$ fiſh, 3 = $\frac{1}{3}$ head
head 9
body 16
32 proof.

What number is that, which being increaſed by $\frac{1}{2}$ and $\frac{1}{4}$ of itſelf and 5 more will be doubled ?

ſuppoſe 8, 8 + 4 + 2 + 5 = 19 − $\overline{8 + 8}$ = 3 + error.
ſuppoſe 24, + 12 + 6 + 5 = 47 − $\overline{24 + 24}$ = 1 − error.
1ſt error 3 x 24 = 72, + 8 x 1 − (2d error) = 80, ÷ $\overline{3 + 1}$ = 20 A.
20 + 10 + 5 + 5 = 40 proof.

EQUATION,

IS the bringing or reducing of ſeveral unequal things to an equality of price, quantity, &c.

EQUATION OF PAYMENTS.

To find a mean time in which ſeveral ſums, received or due at different times, may be paid at once ; and the time

Double Suppoſition is the ſuppoſing of two numbers, quanti- es, &c. from which if both prove falſe, and " the firſt error to the ſecond, as the difference between the true and firſt ppoſed number, is to the difference between the true and cond ſuppoſed number ;" the required number may be und by the following

Rule. Uſe any two convenient numbers as though each ere the true number, and it will be found how much they iffer in themſelves, or from the reſult given.

Multiply each ſuppoſition by the error (or difference) of e other ; and if the ſuppoſitions were both too ſmall or large, ivide the difference of the products by the difference of the rrors : but, if one ſuppoſition were too large,& the other too nall, divide the ſum of the products by the ſum of the er- ors ; in either caſe the quotient will be the anſwer.

When the errors are equal, and ſuppoſitions different, alf the ſum of the ſuppoſitions is the anſwer.

Examples.

« A and B have the ſame income : A ſaves $\frac{1}{8}$ of his ; but , by ſpending 30 £ per annum more than A, at the end of years finds himſelf 40 £ in debt ; what was their income, nd, what does each ſpend per annum ?

```
          Suppoſe income 80 x 1/8=10 -80=70 A's expenſe.
        Suppoſe          160            30
                         20=1/8 of 160  ——
                         ——             100 B's ——
ſup.   error.            140 A's expenſe  8 years
 160x120=19200            30            ——
  80x 40 =3200 -         ——             800
            ——           170 B's   80x8=640(whole)income
20 -40=80)16000          160 inc. yearly ——
  As.       ——           ——             160
early income 200          10             40
's expenſe   175           8 years      ——
              30          —             120 + error.
             ——           80
3's expenſe  205          40
               8          —
              —           40 + error.
    proof 40
```

A merchant having 1200 $ due to him, $\frac{1}{6}$ of which payable in two months, $\frac{1}{3}$ in three months, and the reſt in ſix months; the creditor agreeing to pay $\frac{1}{2}$ in advance, at what time ought the remainder to be paid ? 8$\frac{2}{3}$ m. A.

$\frac{1}{6}+\frac{1}{3}=\frac{3}{6}, -\frac{6}{6}=\frac{3}{6}$ payable in 6 m.
$\frac{2}{6}+1+3=4\frac{1}{3}$, mean time for payment of
the whole ; but $\frac{1}{2}$ being paid in advance,
$4\frac{1}{3}$ m. x*d* by $2=$ A.

$\frac{1}{6}\ x\ 2=\frac{2}{6}$ m.
$\frac{2}{6}\ x\ 3=1$ m.
$\frac{3}{6}\ x\ 6=3$ m.

ALLIGATION OR EQUATION OF QUALITIES,

IS the reducing of ſeveral different qualities and prices to an equivalent ſimple quality.

When the quantities and prices are given ; to find the mean price of the mixture.

Alligation Medial.

RULE. Multiply each quantity by its price, and the ſum of the products is the price of the whole ; divide the whole price by the whole quantity, and the quotient is the mean price of 1 gln, lb, &c of the mixture ; as

If 20 gallons of wine at 1 $ per gln be mixed with 15 glns at 8 dimes per gln, and 7 glns at 125 cents per gln ; what is 1 gln of the mixture worth ?

glns. dimes
20 x 10 = 200
15 x 8 = 120
7 x 12,5 = 87,5
42 42)407,5(9,7di A.

Required the value of the mixture if 8 glns of water were added ? (8 x 0 = 0 value of water in wine, to the buyer)
42 + 8)407,5(8,15di per gln A.

But when the quantities are unlimited, and the prices of the ſimples and mixture given ; to find what quantity of each may be mixed.

Alligation Alternate.

RULE. Subtract the price of the mixture from the price *of the* deareſt ſort, this remainder may be conſidered as the *quantity (to be mixed)* of any cheaper ſort (or ſorts than the *mixture,)* the price of which taken from the mixture price e quantity of ſaid dearer ſort, (to be mixed :) thus proceed ith each dearer and cheaper ſort, conſidering the remainder each as the quantity to be mixed) of the other.*

If the number of dearer or cheaper ſorts exceed the other, e one or more of the oppoſits as often as neceſſary.

If the mixture be too ſmall or large xy or ÷e by ſuch a imber as will bring it to a ſuitable or given quantity ; a xr ay be found by ÷g the given quantity by the quantity und as above.

If one ingredient or more be limited, find the quantities as efore, and xy or divide it and its oppoſit by ſuch a number will bring it to its limited quantity.

EXAMPLES.

f a merchant would mix wines worth 16, 22, 31 & 35 dimes er gallon ſeverally; what quantity of each may be uſed, at the mixture may be worth 25 dimes per gln ?

prices.	glns. prices. values.		gallons.	A.
5 —16=	9 at 35=315	or 16	10+ 6=16 x 2=	32
5 —22=	3 at 31= 93	22	10+ 6=16	32
5 —31=	6 at 22=132	31	9+ 3=12	24
5 —35=	10 at 16=160	35	9+ 3=12	24
5 x	28=(pf.=)700	If a cask of 112 glns. were to be filled, 2 is the xr for the whole.	56)	xr 112(2

A merchant mixes wines worth 14 s 15 s 19 s and 22 s er gallon, ſo that the mixture may be worth 18 s per gln ; hat quantity of each may be mixed ?

				glns.	
8s —14=	4 at 14 s	or 14	4	4 at 14s	As.
8 —15=	1 at 15 s	15	1+ 4=	5 at 15s	
8 —19=	3 at 19 s	19	3	3 at 19s	
8 —22=	4 at 22 s	22	4+ 3=	7 at 22s	

If 8 gallons at 14 s were to be mixed, each quantity, found by the rst operation, may be xd by 2 for the A. or xy the quantity at 22 s by for the answer.

• • • • • •

---

* Because the *greater* the price of one sort the more it requires of *cheaper sort to balance it* : and the cheaper any sort is the more of *earer is necessary.* It is impossible to enumerate all the answers *le admits of.*

How much gold of 15 and 22 carats fine, may be mixed with 4 oz 19 carats fine ; to compoſe a mixture of 34 oz 20 carats fine ?

20 –15= 2, x 2=4, x 2=8 oz of 15 carats
20 –22= 5+1, x 2=12,+ $\overline{5 \times 2}$=22 of 22 carats.
20 –19= 2, x 2=4, oz of 19 carats.

PROGRESSION,

ASCENDING AND DESCENDING,

IS a regular increaſe and decreaſe of number or value by ſome common or increaſing and decreaſing ratio ; and may be divided into three kinds, (all increaſing and decreaſing differently) viz. Arithmetical, Changing and Geometrical Progreſſion.

INVOLUTION.	EVOLUTION.
ARITHMETICAL	PROGRESSION.
Is the increaſing or - - - - - - value by the addition or - - - number ; as,	decreaſing of numbers and - - ſubtraction of ſome equal - -
1, (+1=) 2, (+1=) 3, 4, 5, 6, 7, 8, 9, &c, 3, (+2=) 5, 7, 9, 11, 13, 15, 17.	9, (–1=) 8, 7, 6, 5, 4, 3, 2, 1, 0 17, (–2=) 15, 13, 11, 9, 7, 5, 3
CHANGING PROGRESSION	OR PERMUTATION.
Is progression by an increasing xr to find the number of times any number of things may stand in a different relation ; or - - - - - -	or decreasing divisor ; - - - - - - to find the number of things which may be in any given number of positions ; as,
1 x 2 x 3 x 4 x 5 x 6 x 7 x 8 ‖ ‖ ‖ ‖ ‖ ‖ ‖ ‖ 1, 2, 6, 24, 120, 720, 5040, 40320,	8) 7) 6) 5) 4) 3) 2) 40320(5040(720(120(24(6(2 40320(20160(6720(1680 (336(56(8 2) 3) 4) 5) 6) 7) 8)
GEOMETRICAL	PROGRESSION.
Is the increasing of numbers by xn of some equal number ; i. e. - - - - given number any power, term, - - value the same in a greater - - - - -	or decreasing by the division - - - - the ascertaining from any - - - - - - or root required ; leaving the - - - or less number ; as
1 2 3 4 5 6 7 8 9 2 x 2 x 2 x 2 x 2 2 2 2 2 ‖ ‖ ‖ ‖ ‖ ‖ ‖ ‖ ‖ 2, 4, 8, 16, 32, 64, 128, 256, 512, 3, (x 4=) 12, (x 4=) 48, 192, 768,	2) 9 8 7 6 5 4 3 2 1 512, (256, 128, 64, 32, 16, 8, 4, 2 768, (÷4=) 192, 48, 12, 3.

ARITHMETICAL PROGRESSION.

The two extremes of an arithmetical progreſſion or proortion* are equal to any two means equally diſtant from the xtremes, or equal to double the middle terms; as,

2, 4, 6, 8, 10; 2+ 10=4+ 8, or 6+ 6, (2+ 8=4+ 6)

To ſolve queſtions of this nature eaſily; it is neceſſary hat 3 of the 5 following numbers ſhould be known; viz.

ſt the firſt term
d the laſt term
d the No. of terms
th the common difference
th the ſum of all the terms

The firſt & laſt terms are called extremes, and the terms between them are called means; the common difference is the No. +*d* or −*d*.

Rule 1. Multiply the ſum of the two extremes by half he number of terms, or the whole number of terms by half he two extremes, and the product is the ſum of all the terms.

Ex. How many ſtrokes does the hammer of a clock ſtrike n 12 hours? extremes 1+ 12=13, x (½ 12=) 6=78 A.
2)
or 1+ 12=13 x 12=156(78 times. A.

Rule 2. Divide the difference of the extremes by the common difference, and the quotient increaſed by one is the number of terms.

Ex. If a man gave his youngeſt ſon 400 $, the next 600, &c. increaſing to the eldeſt, who had 2000 $; how many ſons had he?

extr's. 2000 −400=1600, ÷200(com.differ.)=8,+ 1=9A.

Rule 3. Divide the difference of the extremes by one leſs than the number of terms, and the quotient is the common difference.

Ex. In the laſt example the difference of extremes is 1600; what is the common difference? 1600 ÷ $\overline{9-1}$=200A.

. . . . . .

* When numbers increase or decrease arithmetically, the proportio or relation which they bear to each other is arithmetical; so (of 4 term the first term is as much less or greater than the second as the thir less or greater than the fourth, &c. as 2 . . 4 :: 6 . . 8 arithmetical

Rule 4. Multiply the number of terms by the common difference; from the product ſubtract as many as the equal difference exceeds the firſt term, or add as many as the firſt term exceeds it, and the reſult is the laſt term.

Ex. How far would a veſſel ſail the 20th day, if ſhe ſailed 6 miles the firſt day, and increaſed 8 miles each day?

Equal difference 8, x 20 = 160, − 2 = 158 A. (+ 6 x 10 = whole diſtance.)

What would have been the laſt day's ſail, as above, if 15 were the firſt? $\overline{20 \times 8} + \overline{15 - 8} = 167$ A.

PERMUTATION.

Any number of things being given; to find how many changes they will admit of; or how many different poſitions they may be placed in, upon the ſame line.

Rule. Multiply together an arithmetical ſeries of numbers, which increaſes from one by one, to the number of things, and the laſt product is the number of changes required.

If the number of changes be $\div d$ by an increaſing or decreaſing ſeries (of the ſame kind of the above mentioned,) until the largeſt $\div r$ and ſmalleſt $\div nd$ be alike, either of them will be the number of things varied.

Examples.

How many different poſitions may 7, 8 and 9 be placed in on a line? $\underset{7}{1}\times\underset{8}{2}\times\underset{9}{3}=6$ A. 789, 879, 978, 987, 897, 798 proof.

In how many different poſitions, in the ſame path, may half a dozen ſheep walk to a paſture? 1x2x3x4x5x6 = 720 A.

$$\left.\begin{array}{l}\quad\ 3)\ \ 4)\ \ 5)\ 6)\\ 2)720(340(120(30(6\end{array}\right\}\text{proof}=\left\{\begin{array}{l}\text{number of things admit-}\\ \text{ting of 720 changes.}\end{array}\right.$$

How many changes may be rung on 12 bells, and how long would they be ringing once over, if 2 changes were rung in a minute? 479001600 number of changes, or half mn. A. = 455 y. 3150 h. allowing 365 days 6 hours to the year.

GEOMETRICAL PROGRESSION.

When any rank or ſeries of numbers are in geometrical *progreſſion* or proportion; i.e. increaſing or decreaſing by an ual ratio; the product of the two extremes, is equal to e product of any two means equally diſtant from the extremes, and is equal to the ſquare of the middle term; as, 4, 8, 16, 32; 2 x 32=4 x 16=8 x 8=64.

When the firſt term or *root* of a geometrical progreſſion is ven, the power required is repreſented by the index or exnent placed at the right of the given root; as, $2^1, 2^2, 2^3$ d 2^4, repreſent 2 4, 8, and 16; $3^3=27$; $12^2=144$.

But when a *power** is given and its root required, or when number of terms are given, (whether they are powers of the ſt term or not,) the index or exponent is placed at the left top of the given term with or without the radical ſign, ($\surd$;) us $\sqrt[2]{4}$, $\sqrt[3]{8}$, ${}^4 16$, ${}^5 32$ repreſent (the 2nd, 3rd, 4th and 5th ots = 2, 2, 2, 2.

When a number is pointed the points repreſent its power; e firſt point is placed over units, the ſecond over the ſecond gure from units for the ſecond power, over every third fige for the third power, &c. as, ${}^2 1\dot{3}4\dot{2},0\dot{3}$; $\dot{1}27\dot{3}45\dot{6},01\dot{0}$.

The firſt *term* and *ratio* given; to find the other terms. r to involve numbers.

RULE. Multiply the firſt term & ratio together, multiply is product by the ſame ratio, &c. (increaſing the indices acrdingly,) and the products are the ſeveral terms or powers quired; and if the firſt term be equal to the ratio the ſend is the ſquare, the third is the cube, &c. as in the folwing table of powers.

To involve fractions; multiply each part of the given action into itſelf until both arrive at the power required, r the anſwer.

EXAMPLES.

What is the 2d, 3d, 4th, &c. power of 3, of $\frac{2}{3}$ and of ,4?

$$\left.\begin{array}{ll} \overset{1+1=}{x\,3} = \overset{2+1=}{{}^2 9},\ x\,3 = \overset{3+1=4}{{}^3 27\ x\,3} = {}^4 81 & {}^2 9,\ {}^3 27.\ {}^4 81,\ {}^5 243\ \text{ps. of } 2. \\ \overset{+\ 1=2+\ 1=3\ +\ 1=4}{\times\frac{2}{3}=\frac{4}{9},\ \times\frac{2}{3}=\frac{8}{27},\ \times\frac{2}{3}=\frac{16}{81}} & \overset{2}{\frac{4}{9}},\ \overset{3}{\frac{8}{27}},\ \overset{4}{\frac{16}{81}},\ \overset{5}{\frac{32}{243}}\ \text{ps. of } \frac{1}{2}. \\ 4\times{}^1,4 = {}^2,16\times{}^1,4 = {}^3,064 & {}^2,16,\ {}^3,064,\ {}^4 0256\ \text{ps of } 4. \end{array}\right\}\text{As.}$$

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* When the *first term* and *ratio* are equal they are usually called the ot, *and the other terms powers*; but when the first term and ratio *ffer, the several numbers of the* progression are known only by the *me of terms; and if the* 2d, 3d, &c. term be taken as a power, *ot is a number between* the ratio and first term.

If a ſum of money be divided among 6 perſons ſo that the firſt may have 10 $\$$, the ſecond 20, &c. increaſing in a duplicate ratio; what is the ſhare of each?

$$\overset{1}{10},\times 2=\overset{2}{20},\times 2=\overset{3}{40},\times 2=\overset{4}{80},\ \overset{5}{160},\ \overset{6}{320},\ \text{As.}$$

The firſt term and ratio given; to find any term or power required, and the ſum of all the terms.

Rule. Multiply the ratio by itſelf, multiply this product by itſelf, &c. increaſing the arithmetical indices accordingly, until the index repreſents the ratio raiſed to a power one leſs than the term required, multiply this power by the firſt term, add its index, and the product is the term, and the ſum the index required.

Multiply the laſt term by the ratio, from the product ſubtract the firſt term, divide this ſum by the ratio leſſened one, and the quotient is the ſum of all the terms.

Examples.

If a bird flying eaſtward 12 days, went 3 miles the firſt day and tripled her flight each day; what would be the laſt day's flight, and the whole diſtance flown?

$$\begin{array}{l} \quad 1+1=2+2=4\ +\ 4=8+\quad 4=12 \\ 3\times3=9,\times9=81,\times81=6561,\times81=531441\ \text{m. last day A.} \end{array}$$

1st term=ratio,=3

$$\begin{array}{r} 3 \\ \hline 3-1=2)1594323\ \ -3 \\ \hline 797160\ \text{whole dis. A.} \end{array}$$

(24855)531441(21← times round the globe.)

"A ſum of money is to be divided among 8 perſons, the firſt to have 20 £, the ſecond 60 £, and ſo on in triple proportion; what will the laſt have?"

20)60(3 ratio. $\overset{7+1}{3\times 20}=\text{A.}$

$$\begin{array}{l} 1+1=2+1=4+\quad 2=6+\quad 1=7+\quad 1=8 \\ 3\times3=9,\times9=81,\times9=729,\times3=2187,\times20=43740\ \text{£ A.} \end{array}$$

$$\text{or}\ \overset{0}{20}.\ \overset{1}{60}.\ \overset{2}{180}.\ \overset{3}{540},\frac{540\times540}{20}=14580\ \text{then}\ \frac{14580\times60}{20}=\overset{7}{43740}\ \text{£. A.}$$

• • • • • •

The following is the common method of using the indices or exponents *of numbers,* and of finding a remote term from the first term and *ratio given.*

*"When the first term of the series& the ratio are equal, the indices

" A perſon agreed to ſerve a gentleman twelve months, ovided he would give him a farthing for the firſt month's vice, a penny for the ſecond, four pence for the third, &c. iat did his wages amount to ?"

$\overset{1}{4}=\overset{2}{16}, \times \overset{2}{16}=\overset{4}{256}, \times\overset{4}{256}=\overset{8}{65536}, \times\overset{3}{64}=\overset{11+}{4194304}, \times \overset{1=12}{1}=4194304,$

$$\frac{\overset{1+}{}\times \overset{1+}{1} \times \overset{1}{4}+1}{\overset{1}{4}-1}=A \quad \text{ratio between f. \& d.}=4, -1= \quad 3)\overline{16777216}\ -1$$

$$5825\,£\ 8\,s\ 5\tfrac{1}{4}\,d= \quad 5592405\ \text{f A.}$$

. . . . . .

ist begin with an unit, and, in this case, the product of any two terms equal to that term, signified by the sum of their indices.

* But when the first term of the series and the ratio are different, indices must begin with a cipher, and the sum of the indices, made nice of, must be one less than the number of terms, given in the quesn ; because 1 in the indices stands over the second term, and 2 in the lices over the third term, &c. And, in this case, the product of any o terms, divided by the first, is equal to that term beyond the first, nified by the sum of their indices.

$$\text{Thus}\left\{\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & \text{\&c. Indices.} \\ 1 & 3 & 9 & 27 & 81 & 243 & 729 & \text{\&c. Geometrical series."} \end{matrix}\right.$$

" In any Geometrical Progression proceeding from unity, the ratio ing known, to find any remote term, without producing all the interediate terms.

RULE. Find what figures of the indices added together would give the ponent of the term wanted, then multiply the numbers standing unr such exponent into each other, and it will give the term required.

NOTE. When the exponent 1 stands over the second term, the numr of exponents must be 1 less than the number of terms.

In a Geometrical Progression not proceeding from unity ——

RULE. Proceed as in the last only observe that every product must divided by the first term.

x. $\overset{0}{1}, \overset{1}{2}, \overset{2}{4}, \overset{3}{8}, \overset{4}{16}, \quad \overset{4}{16}\times\overset{4}{16}\times\overset{3}{8}=\overset{11}{2048}=$(12 term.)

$\overset{0}{10}, \overset{1}{20}, \overset{2}{40}, \overset{3}{80}, \dfrac{80\times 80}{10}=640, \dfrac{640\times 80}{10}=5120$(=10th term)"

" If the first term be 5, and the ratio 3, what is the 7th term ?

$\overset{1}{15}\ \ \overset{2}{45}\ \ \overset{3+}{135}\times\overset{2+}{45}\times\overset{1}{15}=$ 91125 dividend. $3+2+1=6$ Index of the term beyond 1st, or 7th. The number of terms ultiplied is 3, and $3-1=2$, is the power to which the first term is to raised, therefore, $91125\div 25=3645$, $A.=135\times\frac{45}{5}\times\frac{15}{5}$."

$\overset{1+}{3}\times\overset{1+}{3}\times\overset{1}{3}=\overset{3+}{27}\times\overset{3+}{27}\times\overset{1}{5}=\overset{7}{3645}$ A.

A TABLE OF POWERS.

Roots, or 1st Powers	I	2	3	yd 4 lb	5	6	,7	8 mi	9	E 10	ft. 12lb
Squares, or 2d Powers	I	4	9	qr 16 qr	25	36	,49	64 fur	81	$100	ir.144oz
Cubes, = 3d Powers	I	8	27	na 64 oz	125	216	,343	512	729	d 1000	"1728
Biquadrates, = 4 P.	I	16	81	256 ¼oz	625	1296	,2401	4096	6561	c10000	/"20736
Sursolids, = 5 P.	I	32	243	1024 dr	3125	7776	,16807	32768	59049	100000	248832
Square cubes, = 6 P.	I	64	729	4096	15625	46656	,117649	262144	531441		
2d Sursolids, = 7 P.	I	128	2187	16384	78125	279936	,0823543	2097152	4782969		
Biquadrates sq'd 8 P.	I	256	6561	65536	390625	1679616	,05764801	16777216	43046721		
Cubes Cubed, 9 P.	I	512	19683	262144	1953125	10077696	40353607	134217728	387420489		
Sursolids sq'd, 10 P.	I	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401		

A TABLE OF
CORRESPONDING SERIES, IN
Arithmetical & Geometrical Progression.

Ind.	Nos.		Indices.		Numbers		Ind.
10000	2	31	49542	59542	62	29791	148624
15849	3	32	50000	65849	96	32768	150000
20000	4	33	50444	70444	132	35937	151332
23219	5	34	50874	74093	170	39304	152622
25849	6	35	51289	77138	210	42875	153867
28070	7	36	51699	79769	252	46656	155097
30000	8	37	52094	82094	296	50653	156282
31699	9	38	52479	84178	342	54872	157437
33219	10	39	52834	86053	390	59319	158502
34594	11	40	53219	69188	121	64000	159657
35849	12	41	53572	71698	144	68921	160716
37004	13	42	53919	74008	169	74088	161757
38070	14	43	54213	76140	196	79507	162639
39068	15	44	54594	78136	225	85184	163782
40000	16	45	54918	80000	256	91125	164751
40870	17	46	55235	81740	289	97336	165705
41699	18	47	55546	83398	324	103823	166638
42479	19	48	55849	84958	364	110592	167547
43219	20	49	56140	86438	400	117649	168420
43919	21	50	56439	87838	441	125000	169317
44594	22	51	56723	89188	484	132651	170169
45235	23	52	57204	90470	529	140608	171612
45849	24	53	57296	91698	576	148877	171888
46439	25	54	57548	92878	625	157464	172644
47004	26	55	57813	94008	676	166375	173439
47548	27	56	58070	95096	729	175616	174210
48070	28	57	58328	96140	784	185193	174984
48580	29	58	58580	97160	841	195112	175740
49068	30	59	58829	98136	900	205379	176469

Cube roots of - -

By a Table of corresponding series of numbers adapted to each other, [as those in the above tables according to the preceding rules,] the result of any operation performed according to the progression of its series may be represented by the result of the same term as in a corresponding series performed according to its progression; as, the index 1 in the table of powers represents 2, 3, 4, &c. and the index 2 represents 4, 9, 16, &c. each of which may represent the index 2 & every second power in the table; 9 x 81=72 oppost the pt is 3 (=1+2) & 8 (=2x4

LOGARITHMS

ꝛE the arithmetical indices of a geometrical ſeries of numrs; by which the reſults, from multiplication and diviſion, the geometrical numbers, may be repreſented by the adſion and ſubtraction of thoſe in arithmetical progreſſion; in the preceding tables.

The indices or exponents commonly uſed are, 1 for the mber 10, 2 for 100, 3 for the 3d power of 10, &c.

1 + 1 = 2   3   4 .—   3 =   1   0 .—1 .—2 indices or lgrs.
10, x 10 = 100, 1000, 10000, ÷1000 = 10, 1, ,1 ,01 geometric terms.

To form an arithmetical ſeries correſponding to the interediate numbers from 1 to 10, from 10 to 100, &c.

Rule.* To one of the numbers whoſe index or logarithm required, add the next leſs number whoſe logarithm is own; by their ſum divide (as in integers) the conſtant denal ,8685889638+ (neglecting the remainder,) divide the otient by the ſquare of their ſum, divide this quotient by e ſame ſquare, &c. till the laſt quotient be leſs than its diſor, or be removed as many places to the right as the reired logarithm is to contain figures.

Divide as before the firſt quotient by 1, the ſecond by 3, e third by 5, &c. (increaſing each diviſor by 2.) Add the veral quotients of 1, 3, 5, 7, &c. to the known logarithm f the ſaid number 1 leſs than that of the logarithm requir-,) and the ſum is the index or logarithm of the number for hich it was ſought.

After finding a ſuitable number of logarithms as above; ld together the lgrs. of ſuch numbers as by x*n*, or ſubtract e lgr of the ÷*r* from the lgr. of the ÷*nd* of ſuch numbers

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[*] If we ſuppoſe a ſufficient number of geometrical proportionals beeen 1 and 10 and as many arithmetical indices between 0 (which is e index of 1) and 1, (the index of 10,) there will be indices among em ſufficiently near to each of the intermediate numbers from 1 to 10; , ſuppoſe the number of geometrical proportionals between 1 and 10, d alſo the number of indices between 0 and 1. to be 9999; then the mber of terms, including 1 and 10, will be 10001; and the 3011th ometrical term will be 1,9999 or 2 nearly and its correſponding index ,3010, which is the logarithm or index of 2 nearly. Again the 4772d *ometrical term is 2,9999* or 3 nearly, and the correſponding index is *771, which is the index of* 3 nearly, &c.

as by diviſion, would give the number of the lgr required, or ÷e by 2 the lgr of the ſquare, or by 3 that of the cube, &c. of any number whole root is the number of the lgr required; obſerve the ſame in the uſe of lgrs.

EXAMPLES.

If, 0 be the index of 1, and 1 that of 10; what is the index or lgr. of 2, 3, 4, &c.?

The given number 2,+1 (next leſs number,)=3, and the ſquare of 3=9.

The number 3,+2 (next leſs number,)=5, & 5 x 5=25.

3)868588964 –
9)289529654 ÷1=,289529654+
9) 32169962 ÷3=,010723321 –
9) 3574440 ÷5= 714888+
9) 397160 ÷7= 56737+
9) 44129 ÷9= 4903+
9) 4903 ÷11= 446 –
9) 545 ÷13= 42 –
9) 61 ÷15= 4+
6 ÷17

(l. of1=0+) l.of $\frac{2}{1}$,301029995+
(to l. of $\frac{2}{1}$+) l. of 2 ,301029995$\frac{2}{3}$
(to l.of 2+) l. of 4 ,602059991$\frac{1}{3}$
(to l.of 4+) l. of 8 ,903089987
l. of 4 x 8 1,505149978$\frac{1}{3}$

,5),868588964
,25),173717793 ÷1=,173717793
25) 6948712 ÷3= 2316237
25) 277948 ÷5= 55590
25) 11118 ÷7= 1588
25) 445 ÷9= 50
18 ÷11= 2

lgr. of $\frac{3}{2}$= ,176091260
(to l. of $\frac{3}{2}$+) l. of 2 ,301029995
(to l. of 2+) l. of 3 ,477121255
(from 1 –l.of 2) l.of 5 ,698970004
(to l. of $\frac{3}{2}$+) l. of 6 ,778151250
l. of 9 ,954242510
for l. of 7, ÷e, ,86858 &c. by 6+7, ÷e qt. by 13 x 13, &c.

DESCRIPTION AND USE OF LOGARITHMS.

In uſing logarithms their unit figure only is conſidered as the index.

"*Integral* numbers are ſuppoſed to form a geometrical ſeries, increaſing from unity toward the left; but decimals ar ſuppoſed to form a like ſeries, decreaſing from unity toward the right, and the indices of their logarithms are negative ;" as + 1 is the index of 10, and −1 is the index of $\frac{1}{10}$ or ,1 : + 2 is the index of 100, and −2 is the index of ,01, &c. becauſe multiplying by a decimal is equal to dividing by a whole number, as many times larger than 1, as the decimal is leſs than 1.

Where 1 is the index of 10, the index of any other number is always one leſs than the number of places occupied by the integers of that number and always one more (to be −*d*) than the number of ciphers, at the left, before the number begins, and all numbers whoſe figures are the ſame, have logarithms whoſe decimal parts are the ſame ; as, the logarithm, of 8976 is 3,95308, of 89,76 is 1,95308, of 8,976 is 0,95308, of ,8976 is −1,95308, of ,008976 is −3,95308, &c.

To find, in a table, the logarithm of any given number.

Find the given number, in the table under No. and to the logarithm, in the next column oppoſit, prefix the proper index of the given number for the anſwer ; if the number given contain more figures than any in the column of numbers, find the next figure in the upper line of numbers, under which, oppoſit the number firſt found, is the decimal part of the logarithm required, to which prefix its index ; if the *number* thus found do not contain a ſufficient number of figures, ſubtract it from the next larger number, then as this difference is to the difference of their correſponding logarithms, ſo is the remaining part of the given number to its part of the logarithm required, which add (to the leſs lgr. for the anſwer.

What is the logarithm of the number 63,975 ?

l. of 63970 is 80598	As 10 : 6 :: 5 : 3 to be
l. of 63980 is 80604	added to 80598, their ſum with
their dif. is 10 & 6	the index is 1,80601 A. lgr to
	5 places.

To find in a table the natural number to any given logarithm.

Find the given logarithm among the logarithms in the ta-le, and its correſponding number as before ; which point according to its index for the anſwer : if the logarithm cannot be found in the table ; as the difference between the next larger and ſmaller logarithms is to the difference between their correſponding numbers, ſo is the difference between the given lgr. and the next leſs lgr. in the table to its proportional part of the number required, which add to the number of ſaid leſs logarithm for the anſwer.

What is the number to the logarithm 1,5326606 ?

Next larger *l.* is 5326652 its No is 3409300 ; given *l.* 5326606
Next leſs *l.* is 5326525 its No. is 3409200 ; leſs *l.* 5326525
Differences - - - - 127 - - - - - - - 100 - - - - - - - - - 81

As 127 : 100 :: 81 : 64 nearly, to be added to the No. of the leſs *l.* for the No. required, which is to contain two places of integers, or one more than the given index 1,) ſo 34,09264 is the A. If the given *l.* were —1,5326606, its correſponding number would be ,3409264, if the index were —2, the A. would be ,034 &c. and if 2, the A. would be 340,9 &c.

The SUM *of the Logarithms of any two or more numbers is the Logarithm of the* PRODUCT *of thoſe numbers.*

To add logarithms ; proceed as in ſimple addition, when their ſigns are ſimilar, i.e. all affirmative ; as, +1, +2, or 1, 2 ; or all negative ; as, —1, —2, or —1, —2, —4 : but if their ſigns are diſſimilar ; as, +1, —2, add the decimals, which are always affirmative, and the difference, between the negative and affirmative indices with the ſign of the greater, is their index.

Multiply 275 by 12,6.		Multiply 275 by ,00126	
To *l.* (of 275 viz.)	2,43933	To *l.* of 275,	+2,43933
add *l.* (of 12,6	1,10037	Add *l.* of ,00126,	—3,10037
ſum (is *l.* of A.)	3,53970	their ſum is *l.* of pt.	—1,53970

Multiply 2750, by 1260. The *l.* of the product is 6,53970. The tabular numbers correſponding to the above logarithms, are 3465, (to *l.* 3,5397,) ,3465, (to *l.* —1,5397,) 3465000, (to *l.* 6,5397)

Subtract the logarithm of the DIVISOR *from that of the* DIVIDEND, *and the remainder is the logarithm of the* QUOTIENT.

To ſubtract logarithms; proceed as in ſimple ſubtraction, when their ſigns are ſimilar, and the ſubtrahend leſs than the minuend: but if the dicimals of the ſubtrahend exceed; add 1 to its index, if it be affirmative, if negative ſubtract 1; then change its ſign, and the difference between the two indices with the ſign of the greater is the index of the logarithm required.

Divide 24163 by 4567. ÷*nd* 24163 its *l.* 4,3831509 min.
,7235199 *l.* of A. ÷*r* 4567 its *l.* 3,6596310 ſub.
qt. 5,295782 *l.* 0,7235199 dif.

Divide 37,149 by 523,76.
l. of ÷*nd* 1,5699471
l. of ÷*r* 2,7191323
l. of qt. −2,8508148

1 added to the index of the ÷r's lgr & its affirmative sign changed to negative becomes −3, and the dif. of the indices is 2, the sign of the greater (or 3) is −.

Divide ,06314 by ,007241.
l. of ÷*nd* is −2,8003046
l. of ÷*r* is −3,8597985
l. of qt. or A. 0,9405061

Here (as above) the decimals of the subtrahend exceed those of the minuend, and the index −3 being negative, 1 is −d from it.

The logarithm of any given ROOT *multiplied by its* INDEX, *produces the logarithm of the* POWER *required.*

To multiply logarithms; multiply as in ſimple multiplication, when the indices are not negative; but if the index be negative, multiply the decimals as before, and if there be an index to the product it is affirmative, and the difference, between this index and the product of the negative index, is the index required.

What is the ſquare of 2,5791? *l.* of root ,4114682
index of the ſquare 2
6,651756 power required its *l.* is - - - - - ,8221364

What is the third power or cube of 3,07146?
l. of root (3,07146),4873449,x3=1,4620347 *l.* of 28,97575 A

What is the fourth power or biquadrate of ,09163?
l. of root − 2,9620377, x 4=3,8481508 pt of the decimals.
−2, x 4= −8, pt of the negative index
the dif. of indices is 5 −5,8481508 l. of ,00000704984

Divide the logarithm of any given POWER *by its* INDEX, *and the quotient is the logarithm of the* ROOT *required.*

To divide logarithms; Divide as in ſimple diviſion, when the given index is affirmative, or when it is negative and diviſible without a remainder; if not, add ſuch a number to the index as will render it diviſible, alſo prefix the ſame number as an affirmative index to the decimal part of the given logarithm (or ſuppoſe the number twice added to the given index,) divide as before, prefix the given ſign and the quotient is the logarithm required.

What is the ſquare root of 365?
l. of ſq're 2,5622929, $\div 2 =$ 1,2811465 *l.* of root, 19,10498 A.

What is the third root of 12345?
l. of cube 4,0914911, $\div 3 =$ 1,3638304 *l.* of root, 23,11162 A

What is the 10th root of 2?
,3010300 $\div 10 =$ *l.* of 1,071773 A.

What is the ſquare root of ,093?
-2,9684829 $\div 2 =$ *l.* of ,304959 A.

What is the cube root of ,00048?
l. of ,00048 is -4,6812412 $\overline{-4+-2} \div 3 =$ -2 index required, $\overline{2+,6812412} \div 3 =$ 8937471 decimal parts required.
or 3)8,6812412(-2,8937471 *l.* of A. ,07829735.

Here -4 not being exactly diviſible by 3, 2 is added, 2 is alſo added to the decimal part of the lgr. and the ſum of their qts. is the lgr. required.

EVOLUTION,

IS the finding of ſuch a leſs number from a greater, as being involved a given number of times, produces the number given; or is the extracting of the firſt term & ratio from the laſt term only. When any *power* is given to find its *root* [or firſt term and ratio] the progreſſion is called evolution or extraction.

To ſhow the number of figures which the required root is to contain, &c. the given power is commonly pointed into periods according to its index:
i.e. a point placed over units, and if the given power be the

ſecond whoſe index is $\frac{1}{2}$, point every ſecond figure from units, both in integers and decimals, if the given number be the 3d, 4th, 5th, &c. power, point every 3d, 4th, 5th, &c. figure from units; as, $\overset{1}{}\dot{2}\dot{3},\dot{1}$; $\sqrt[2]{\dot{5}3\dot{3},6\dot{1}}$; $\sqrt[3]{21\dot{4}02\dot{3}41\dot{2},21\dot{3}}$; $^{4}\dot{2}1462\dot{3}41\dot{2},213\dot{0}$; $^{5}2\dot{7}3431\dot{4},2000\dot{0}$.

EXTRACTION OF THE SQUARE ROOT.

Rule. After the given number is pointed; place a figure in the quotient whoſe *ſquare* is the neareſt to, (but muſt not exceed the figures of,) the firſt left period, ſubtract it therefrom; to the remainder bring down the next period for a dividend; double the quotient figure for the left part of the diviſor, and generally the number of times (or one leſs) that this part of the diviſor is contained in the dividend, except the right hand figure, is the next figure of the diviſor and quotient alſo; Multiply the diviſor by the right hand figure of the quotient, and ſubtract the product from the dividend; to this remainder, (which muſt never exceed the diviſor more than 9) bring down the next period, double the quotient, find the reſt of the diviſor, and proceed as before, and the quotient is the root required; (and is a $\div r$ and qt. or x*nd* and x*r*, and the given power is its pt. or $\div$*nd*.)

To extract the root of a vulgar fraction.

Reduce the fraction to its higheſt or ſhorteſt terms, and extract the root of both the diviſor and dividend for a new one of each; or multiply the diviſor and dividend together, and extract the root of the product for a diviſor to the given dividend, or for a dividend to the given diviſor; or reduce the fraction to a decimal and extract its root.

Note. Whole periods of ciphers, for decimals, may be annexed to any power or remainder, and a decimal placed in the quotient for each period of decimals.

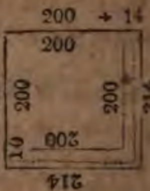
The ſquare *root* of any plane or ſurface is equal to the ſide of a ſquare of the ſame contents. The ſquare root of the *product of any two numbers is a mean proportional between thoſe numbers.*

EXAMPLES.

What is the ſide of a ſquare containing 45796 rods ?

45796(214 A.
$2^2=4$
÷r 41)057
41
÷r 424)1696
1696

45796(200
$200^2=40000$
410) 5796(10
4100
[424)1696(4
1696 —
214)45796(214 214

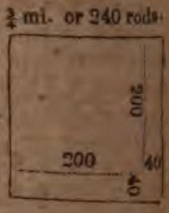


or ſuppoſe 200)45796($\overline{228+200}$ ÷ 2=214 A. neglecting the fractions.

What is the ſide of a ſquare conttaining $\frac{162}{288}$ of a ſquare mile ?

$\frac{162}{288}=\frac{81}{144}$(12 ÷r
1
22) 44

81(9 ÷*nd* $\frac{9}{12}$ A.
81
$\frac{9}{12}=\frac{3}{4}$ mi=240 rods.



or 162
288)162,0000(,5625(,75 A
49
145)725
725

or 81 x 144=11664($\frac{108}{144}$ A.=$\frac{81}{108}$=$\frac{3}{4}$

or $\frac{162}{288}$mi. x 640 acres x 4 rood. x 40 rod = 57600(240 rods A.

. . . .

When there is a remainder after finding a ſufficient number of figures of the root, half this remainder placed over the root, or the whole −r placed over double the root, will be a fraction nearly equivalent to the −r ; if the root with this fraction annexed be squared it will produce a number, the square of the fraction too large, so if 1 or more be added to the divisor of such a fraction it will be generally nearer than the whole fraction.

Wha is the side of a square equal to a circle containing 51 square [illegible]

[illegible] (7½--A.

What is the mean proportional between 24 and 96 ?

$\sqrt[2]{96 \times 24} = \overset{2}{\sqrt{}}2304 = 48$ A. geomet. mean. As 24 : 48 :: 48 : 96.

What is the diameter of a circle containing three times as much as that, whoſe diameter is 4 ?

$\sqrt[2]{4 \times 4 \times 3} = 6{,}928+$ A.

The areas of similar figures are in proportion to each other. As the squares of their similar dimensions. $6{,}928\overset{2}{+} = \overset{2}{4} \times 3$.

If two ſhips ſail from the ſame port, one north 76 leagues, the other eaſt 45 leagues ; how far will they be aſunder ?

$\overline{45 \times 45} + \overline{76 \times 76} = 7801$. $\sqrt[2]{7801} = 88{,}32$ A.

The sum of the squares of the base and perpendicular being equal to the square of the hypothenuse, its root extracted is the distance required. The distances the vessels sail are the base and perpendicular, their distances asunder is the hypothenuse.

What is the ſide of a ſquare equal to a circle whoſe area is 24 ft 4′ 3″ 6‴ ?

```
   ·     ·                                        · 2 3
  24 4′ 3″ 6‴ (4ft 11′ 2″ 7‴+ A.     or 5)24 4 3 6(4 10 5 5
  16                                       20      5
  ——                                       —      ————
8 11′)8 4 3                                 4 4  2)9 10 5 5
      8 2 1                                 4 2    ————
     ————                                   —      4 11 2 8
9 10 2″)2 2 6 0⁗                             2 3
        1 7 8 4                              2 1
       ————                                  —
9 10 4 7‴)6 9 8 0 0⁗                          2 6
          5 9 0 8 1                           2 1
                                              —
                                              5 —r
```

“COMMON METHOD OF EXTRACTING THE CUBE ROOT.

“RULE 1. Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

2. Find the left hand period and put its root in the qt.

3. Subtract the cube, thus found, from the ſaid period, and to the remainder bring down the next period, and call this the *dividend*.

4. Multiply the ſquare of the quotient by 300, calling it the triple ſquare, and the quotient by 30, calling it the triple quotient, and the ſum of theſe call the *diviſor.*

5. Seek how often the diviſor may be had in the dividend, and place the reſult in the quotient.

6. Multiply the triple ſquare by the laſt quotient figure, and write the product under this dividend; multiply the ſquare of the laſt quotient figure by the triple quotient, and place this product under the laſt; under all, ſet the cube of the laſt quotient figure, and call their ſum the *ſubtrahend.*

7. Subtract the ſubtrahend from the dividend, & to the remainder bring down the next period for a new dividend, with which proceed as before, & ſo on till the whole be finiſhed."

"Note. *The ſame rule muſt be obſerved for continuing the operation, and pointing for decimals as in the ſquare root.*"

EXAMPLES.

"Required the cube root of 436036824287?

436036824287(7583 the root.
343

1ſt diviſor= 14910) 93036=1ſt dividend.

73500
5250
125

78875=1ſt ſubtrahend.

2d diviſor= 1687750)14161824=2d dividend.

13500000
144000
512

13644512=2d ſubtrahend.

3d diviſor= 172391940)517312287=3d dividend.

517107600
204660
27

517312287=3d ſubtrahend.

The method of operation.

7 x 7 x 300 = 14700 = 1st triple square.
7 x 30 = 210 = 1st triple quotient.
14910 = 1st divisor.

14700 x 5 = 73500
5 x 5 x 210 = 5250
5 x 5 x 5 = 125
78875 = 1st subtrahend.

75 x 75 x 300 = 1687500 = 2d triple square.
75 x 30 = 2250 = 2d triple quotient.
1689750 = 2d divisor.

1687500 x 8 = 13500000
2250 x 8 x 8 = 144000
8 x 8 x 8 = 512
13644512 = 2d subtrahend.

758 x 758 x 300 = 172369200 = 3d triple square.
758 x 30 = 22740 = 3d triple quotient.
172391940 = 3d divisor.

172369200 x 3 = 517107600
22740 x 3 x 3 = 204660
3 x 3 x 3 = 27
517312287 = 3d subtrahend.

Or, find the firſt figure of the root by the table of powers, r otherwiſe) ſubtract its cube from the firſt given period to e —r bring down the next figure, and divide this ſum by e difference between the cube of the firſt figure of the root d the next larger figure; viz. 8, and the quotient is the d figure of the root; uſe thoſe two figures for the ſuppoſ- d root; as,

x 8 x 8 = 512 436036, &c.
x 7 x 7 = 343 - - - - - - - 343
ifference 169 - - - - - - 169)930(5 2d figure of root.

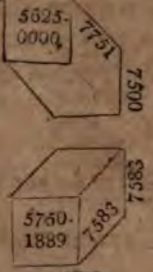
. . . .
ſuppoſe 7500, x 7500=56250000)436036824287(7751
7500 x 7500 =
7500

3) 251

83

A. 7583 root required.

If the breadth & depth of the given solid be 7500, its length must be 7751+, which exceeds the breadth or depth 251, ⅓ of which added to the breadth & to the depth, (⅔ taken from the length) makes them all equal & reduces the figure to a cube, whose dimensions, &c. are as follows



breadth. depth. area. solidity. length.
& ſuppoſe 7583, x 7583=57501889)436036824287(7583 A and proof to the above operation.

"A CONCISE METHOD OF EXTRACTING THE CUBE ROOT.

"RULE 1. Point every third figure of the cube given beginning at the unit's place, then find the nearest cube to the first point, and subtract it therefrom, put the root in the quotient, bring down the figures in the next point to the remainder for a resolvend.

2. Square the quotient and triple the square for a divisor—as, 4 x 4 x 3=48. Find how often it is contained in the resolvend, rejecting units and tens, and put the answer in the quotient.

3. Square the last figure in the quotient, and put it on the right hand of the divisor:

As 6 x 6=36 put to the divisor 48=4836.

4. Triple the last figure in the quotient, and multiply by the former, put it under the other, units under the tens, add them together, and multiply the sum by the last figure in the quotient, subtract that product from the resolvend, bring down the next point and proceed as before.

EXAMPLES.

What is the cube root of 99252847 ?

Square of 4 x 3=48 divisor
Square of 6 put to 48=4836
6 x 3 x 4=72
5556 x 6 = 33336
Square of 46=2116 x 3=6348 divisor
Square of 3=9 put to 6348=*634809
3 x 3 x 46= 414
638949 x 3 = 1916847."

. . .
99252847(463
64
35252
33336
1916847

* * * * * * *

"*When the quotient is 1, 2 or 3, there must be a cypher put to supply the place of tens."

What is the ſide of a cube containing 10345 cubic ft ?

```
                    10345(21,79 — ft.  A.
      20 x 20 x 20 = 8000
   20 x 20 x 3 = 1200\  ——
     1 x 1  =       1 | 2345
   20 x 1 x 3 =    60/
```

What is the cube root of 10345,898 ? 21,79. A.

```
   ſuppoſe 22,                    22,000
          22 x 22 = 484,10345,898(21,375
                                 ———————
                                  3),624
                                     208
                                  ——————
                                  21,792 —A.
```

Multiply the ſolid contents of any number, and the cube root of this product is the ſide of a cube containing ſo many times more (or leſs) than the given cube, as the x*r* is more (or leſs) than 1.

What is the ſide of a cube containing 10 buſhels ?

2150,4 x 10 = 21504,(27,8+ inches A. each ſide of a cube of 10 buſhels.

If the diameter of a globe or ball weighing 20 lbs be 3½ inches; what is the diameter of that weighing 8 times as much of the ſame quality ?

4 x 4 x 4 = 64 x 8 = 512 (8 in. diameter A. wt. 160 lb.

The ſolid contents of ſimilar figures are in proportion to each other, as the cubes of their ſimilar dimenſions.

" If a globe of ſilver of 3 inches diameter, be worth 45 £. what is the value of another globe of a foot diameter ?"
3 x 3 x 3 = 27. 12 x 12 x 12 = 1728. As 27 : 45 :: 1728 : 2880 £. A.

"If a cube of ſilver, whoſe ſide is 4 inches, be worth 50£. I demand the ſide of a cube of the like ſilver, whoſe value ſhall be four times as much?"

4 x 4 x 4=64, & 64 x 4=256. $\sqrt[3]{256}=6,349+$ inches, A.

"Suppoſe a ſhip of 500 tuns has 80 feet keel, 36 feet beam, and is 16 feet deep in the hold; what are the dimenſions of a ſhip of 200 tuns, of the ſame mould and ſhape?"

89 x 89 x 89= 704969=cubed keel.

As 500 : 200 :: 704969 : 281987,6 cube of the required keel.

$\sqrt[3]{281987,6}=65,57$ feet the required keel.

As 89 : 65,57 :: 36 : 26,522=26½ feet beam, nearly.

As 89 : 65,57 :: 16 : 11,7 feet, depth of the hold.

"A GENERAL RULE

"FOR EXTRACTING THE ROOTS OF ALL POWERS.

"1. Prepare the number given for extraction, by pointing off from the unit's place, as the root required directs.

2. Find the firſt figure in the root, by the table of powers, which ſubtract from the given number.

3. Bring down the firſt figure in the next point to the remainder, and call it the dividend.

4. Involve the root into the next inferior power to that which is given; multiply it by the given power, and call it the diviſor.

5. Find a quotient figure by common diviſion, and annex it to the root; then involve the whole root into the given power, and call that the ſubtrahend.

6. Subtract that number from as many points of the given power as is brought down, beginning at the loweſt place, and to the remainder bring down the firſt figure of the next point for a new dividend.

7. Find a new diviſor, and proceed in all reſpects as before."

The BIQUADRATE root may be found, by extracting the *ſquare root*, from the ſquare root of the given number; the SIXTH root, by extracting the cube root from the ſquare root of the given number, &c.

EXAMPLES.

What is the square root of 141376 ?

141376(376
9 = 90000 =

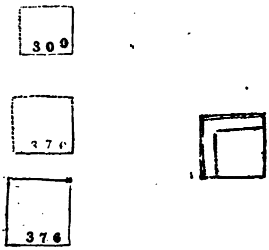
6)51 dividend*

1369 subtrahend = 136900 =

74) 447 dividend

141376 subtrahend =
3 x 2 = 6 divisor
37 x 37 = 1369 subtrahend
37 x 2 = 74 divisor
376 x 376 = 141376 subtrahend.

SQUARES of the Root & its parts.



What is the cube root of 53157376 ?

53157376(376
27 = 37000000

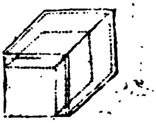
27)261 dividend*

50653 subtrahend = 50653000

4107)25043 dividend

53157376 subtrahend.
3 x 3 x 3 = 27 divisor
37 x 37 x 37 = 50653 subtrahend
37 x 37 x 3 = 4107 divisor
376 x 376 x 376 = 53157376 subtrahend

CUBE.



The natural quotient, of each dividend marked thus, * would considerably exceed the figure required; therefore it is generally best to try a figure 1 or 2 less than the qt. for the 2d figure of the root.

What is the biquadrate root of 19987173376 ?

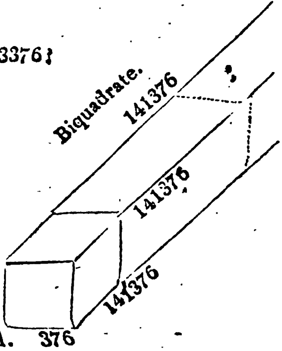
19987173376(376
81

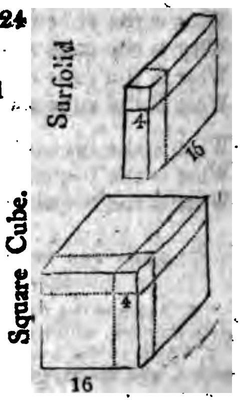
108)1188 dividend*

1874161 subtrahend

202612)11245563 dividend

19987173376 subtrahend
3 x 3 x 3 x 4 = 108 divisor
37 x 37 x 37 x 37 = 1874161 subtrahend.
37 x 37 x 37 x 4 = 202612 divisor
376 x 376 x 376 x 376 = 19987173376 subtrahend





What is the ſurſolid or 5th root of 1024 and of 161051 ? 4 root of 1024

161051(11 root of 161051
$10^5 =$ 100000

$10^4 =$10000)61051*(1

$11^5 =$ 161051

What is ſquare cube root of 4096 ?
A. 4.

The 1ſt, 2d, 3d, 4th, &c. root of any quotient is a common multiplier (or diviſor) between the diviſor and dividend; ſo if the diviſor be multiplied by this root, this product by the ſame root, &c. the ſeveral products are ſo many mean proportionals between the diviſor and dividend, as the index of the root is more than 1; (or if the ÷*nd* be ÷*d* by it the ÷qts are the mean proportionals.)

What is the common multiplier between 4 and 16, which will produce one intermediate term ?

4)16(4(2 A. the ſqr. root of the qt. 4 x 2=8 proportional.

What are the two means between 4,41 and 40,84101 ?

4,41)40,84101(9,261(2,1 cube root. (4,41x 2,1=)9,261 and 19,4481 A.

What are the 3 means between 4 and 324 ?

3) 3) 3)
4)324(^{4}81(3 3)324(108(36(12(4 (as 4 :) 12 :: 36 : 108 (: 324) A.

What is the ſquare root of 106929 ? A. 327
What is the ſquare root of $\frac{275}{341}$? A. ,89802+
What is the ſquare root of $51\frac{21}{25}$? A. $7\frac{1}{5}$
What is the cube root of 22069810125 ? A. 2805

What is the cube root of 33,230979637 ?	A. 3,215+
What is the cube root of 15926,972504 ?	A. 25,16+
What is the cube root of $\frac{324}{1500}$?	A. $\frac{3}{5}$
What is the cube root of $12\frac{19}{27}$?	A. $2\frac{1}{3}$
What is the biquadrate root of 5719140625 ?	A. 275
What is the biquadrate root of 33362176 ?	A. 76
What is the biquadrate root of 531441 ?	A. 27

"RULE FOR EXTRACTING THE ROOTS OF ALL POWERS, BY APPROXIMATION.

"1. Assume the root nearly, and raise it to the same power with the given number, which call the assumed power.

2. Then, as the sum of the assumed power multiplied by the index more 1, and the given number multiplied by the index less 1, is to the sum of the given number multiplied by the index more 1 and the assumed power multiplied by the index less 1, so is the assumed root to the required root.

Or, as half the first sum is to the difference between the given and assumed powers, so is the assumed root to the difference between the true and assumed roots; which difference added or subtracted gives the true root nearly.

And the operation may be repeated as often as we please by using always the last found root as the assumed root, and its power as aforesaid for the assumed power."

"Required the fifth root of 21035,8. Here it appears, that the 5th root is between 7,3 and 7,4. 7,3 being taken, its 5th power is 20730,71593. Hence then

given No. 21035,8 −20730,716 (assumed power)=305,084 difference.

$$5=\text{index};\ 5+1=6;\ 5-1=4;\ 6\div2=3;\ 4\div2=2 \qquad \frac{20730{,}716 \times 3 \quad 21035{,}8 \times 2}{62192{,}148+42071{,}6=104263{,}748}\ (=\tfrac{1}{2}\text{ the first sum}$$

104263,7 : 305,084 :: 7,3 : ,0213605 (dif)+7,3=7,321360=root, true to the last figure."

The first figure of any root may be easily ascertained by a table of powers or by trial; and a figure near the second may be found by dividing the difference between the given power of the first figure and the first point of the given number, with another figure brought down, by the difference between the given power (which will be the 2d, 3d, or 4th, &c.) of the first figure, and the same power of the next larger figure; it is generally best to use a figure 1 larger than this quotient, for the 2d supposed figure; then by annexing a competent number of ciphers to the figures thus found, and involving the result [or supposed root] to a power one less than the given, &c. as by the rule page 64,* al-most any root may be found to 6 or 8 places from two operations; the first three figures will be found at the first trial and the fourth will not be more than 1 too large, if the two first figures of the root be used as the supposed number, or if the second supposed figure be 1 too large; by using the first 3 figures, thus found, as the supposed number, 6 figures of the root may be ascertained, at the second operation, and if the first 4 figures be used as the supposed number, 8 figures of the root may be found, &c.

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* *This may be called a method by* APPROXIMATION, as all others are, *when the* ROOT *is* a surd number, which can never be found, a

Required the side of a cubic box containing 25 bushels or 53760 cubic inches?

53760(37 — 64 cube or given power of 4 next larger figure than 1st
3x3x3=27 — 27 cube or given power of 3 first figure.

dif. 37)267(7 — 37 dif.
259

$37 \times 37 = 1369$)53760(39,26 first qt
supposed No. 37,00

3)2,26 dif.

,75
37,00

root nearly 37,75 − 1

$37{,}74^2$)53760(37,7446557
3)
37,74 + ,0015519 = A nearly.

Given No. ÷ last root^2 &c. = 37,7415518181062 = 16 figures of root.

What is the sursolid root of 307682821106715625?

$3100^4 = 923521$)307682821106 &c. (3331 +
30768, &c.
1st fig of root 3; $3^5 =$ 243
dif. between 3^5 & 4^5 is 779)646(1, 2d figure.

3331
3100
5)231
46 +

46 − 1 = 45, + 3100 = 3145 A. 3145^4)307682821106715625(3145 A & proof

What is the 25th root of 33980640,2176483354639275l8?
2 first figure of root.
$2^{24} = 16777216$)33980640,217648335463927518(2,002540
2,000000
given index 25) 2540(101

2, + ,00101 = root nearly.

24th power nearly:
$2{,}00101^{24} = 16981742{,}762$)33980640,2 &c. (2,00101018333 +
2,00101
25) ,00000018333 dif.

,0000000733 qt. } A.
2,00101 + } nearly

* * * * * *

though it be carried to any assignable degree of exactness is still but *an approximation* to the true root; but if the root be not a surd number, any method that will approximate toward a surd will give the perfect answer.

What is the sixth root of 4357283810092678098889764416?
27354 A.

34487717467307513182492153794673,=32017.

1121016281320476236246497942460481,=13527.

ANNUITIES.

An Annuity is a ſum of money payable every year, for any number of years. If the payment be withheld after it becomes due the annuity is ſaid to be in *arrears;* the ſum of all the annuities, with their intereſt while it is in *arrears*, is called the *amount;* and ſuch a ſum as will pay for this amount before it becomes due, is called the *preſent worth.*

To find the amount of an Annuity at Simple Intereſt.

Rule. Multiply one year's *intereſt* of the annuity, by the *ſum*, of an arithmetical ſeries of numbers increaſing from 1 by 1 to the number of years, after the firſt annuity becomes due, and the product is the whole intereſt, to which add all the annuities (or the product of one payment multiplied by the given number of years,) and the ſum is the amount required.

What is the amount of an annuity of 50 $ for 7 years, at 5 per cent, ſimple intereſt?

50 x ,05=2,5 intereſt of 50 for 1 year.
1+6,=7,x6=42, ÷2=21 (=1+2+3+4+5+6.)

52,5 int. of 50 for 21 years.
50 x 7=350, = 50 $ paid at the end of each year.

A. 402,5 amount of 7 payments of 50$ & int.

The interest on the first payment (of 50 $) commences at the end of the first year, and continues to the last year, [6 years;] the interest on the second payment begins at the end of the second year and continues 5 years, &c. to the last payment, which draws no interest.

If a penſion of 600 $ per annum be forborne 5 years, what will it amount to allowing 4 per cent ſimple intereſt?
1+2+3+4=10, x 24 (int. of 600 1 yr.)=240, + 5 x 600=3240 A.

N 2

To find the preſent worth of an Annuity at Simple Intereſt.

RULE. Find (as in diſcount) the preſent worth of each payment, from the time it becomes due, to the time at which its preſent worth is required, and their ſum is the anſwer.

What is the preſent worth of an annuity of 100 $ for 3 years, at 6 per cent, ſimple intereſt ?

As $\overline{100+6}$: 100 :: 100 : 94,3396 preſent worth for 1ſt year
100+12 : 100 :: 100 : 89,2857 ———— 2d year
100+18 : 100 :: 100 : 84,7457 ———— 3d year
A. $ 268,3710 ——— of the annuity.

What is the preſent worth of a yearly payment or annuity of 500 £ to continue 4 years at 5 per cent, ſimple intereſt?
A. 1782 £ 3 s 8½ d.

To find the amount of an Annuity at Compound Intereſt.

"RULE 1.—Make 1 the firſt term of a Geometrical Progreſſion, and the amount of 1 £ for 1 year at the given rate per cent. the ratio.

2. Carry the ſeries to as many terms as the number of years, and find its ſum.

3. Mulſiply the ſum thus found, by the given annuity, and the product will be the amount ſought.

Or multiply the amount of 1£ for 1 year into itſelf so many times as there are years, less by 1; then multiply this product by the annuity; and subtract the annuity therefrom; lastly, divide the remainder by the ratio less 1, and the quotient will be the amount."

What will an annuity of 60 £ per annum, payable yearly, amount to in 4 years, at 6 £ per cent ?

$$1+1{,}06+1{,}06^2+1{,}06^3 \times 60 = 262\,\pounds.\ 9\text{ s. } 6\tfrac{1}{4}\text{ d. A.}$$

$$\text{or } \frac{\overline{1{,}06 \times 1{,}06 \times 1{,}06 \times 106 \times 60}-60}{\overline{1{,}60-1}} = 262{,}49.\text{A.}$$

"*To find the preſent worth of Annuities at Compound Intereſt.*

"RULE 1. Divide the annuity by the ratio, or the amount of 1£ for 1 year, and the quotient will be the preſent worth of the firſt year's annuity.

2. Divide the annuity by the ſquare of the ratio, and the quotient will be the preſent worth for two years.

3. In like manner, find the preſent worth of each year by itſelf, and the ſum of all theſe will be the preſent value of the annunity ſought.

Or, divide the annuity, &c. by that power of the ratio signified by the number of years, and subtract the quotient from the annuity: This remainder being divided by the ratio less 1, the quotient will be the present worth.

The amount of an annuity may also be found for years and parts of a year, thus:—Find the amount for the whole years, as before. Find the interest of that amount for the given parts of a year. Add this interest to the former account, and it will give the whole amount required.

The present worth of an annnity for years and parts of a year, may be found thus:—Find the present worth for the whole years, as before:—Find the present worth of this present worth, discounting for the given parts of a year, and it will be the whole present worth required."

What is the preſent worth of an annuity of 40 £ to continue 3 years, diſcounting at 5 per cent per year, compound intereſt?

ratio = 1,05)40(38,095 preſent worth for 1ſt year.
$\overline{\text{ratio}}|^2$ = 1,1025)40(36,281 ———— 2d year.
$\overline{\text{ratio}}|^3$ = 1,151525)40(34,556 ———— 3d year.
A. 108,932 £ ——— of the whole annuity.

What ready money will purchaſe an annuity of 60 $ to continue 4 years, at 6 per cent, compound intereſt?

$$\frac{60}{1{,}06^4}=47{,}525,\ 60-47{,}525=12{,}475$$

$$\text{And } \frac{12{,}475}{1-1{,}06}=207{,}916\ \text{A.}$$

To find the preſent worth of an Annuity, or Freehold Eſtate to continue forever, at Compound Intereſt.

RULE. Divide the annuity by the given intereſt of 1 for 1 year and the quotient is the preſent worth required.

What is the preſent worth of a perpetual annuity of 40 $ diſcounting at 5 per cent, compound intereſt ?

40 ÷,05(given intereſt of 1)=800 A.
or as 5 : 100 :: 40 $: 800 $ A. whoſe int. or annuity is 40.

If the yearly income of an eſtate be 79 £ 4 s ; what is the value of the eſtate, at 4½ per cent, compound intereſt ?

79 £ 4 s ÷,045=A.
or 79£ 4 s=79,2; x $\frac{1000}{45}$=1760£ A. its annuity or int. is 79£. 4s.

To find the preſent worth of an Annuity or Freehold Eſtate, in reverſion, at Compound Intereſt.

Rule. Raiſe the ratio (or amount of 1 for 1 year) to a power whoſe index is equal to the time in reverſion (or number of years to come before the annuity commences,) multiply this power by the given intereſt of 1 for 1 year; by the product divide the given annuity, and the quotient is the preſent worth required.

“ The reverſion of a freehold eſtate of 79 £ 4 s per annum, to commence 7 years hence, is to be ſold : what is it worth in ready money, allowing the purchaſer 4½ per cent. for his money?$_{7}$

ratio 1,045 x ,045=,06123879)79,2(1293,297 £ A.= preſent worth of 1760 £ due 7 years hence, at the given rate.

What is the preſent worth of an annuity of 60 $ to commence in 3 years, and continue forever, at 6 per cent ?

ratio 1,06,^{3} x ,06 =,07146096)60,(839,615+ A. preſent worth of 1000, due 3 years hence.

TABLE 1. The amount of 1 $, £, &c. at 5 and 6 pr ct. compound interest. from 1 to 30 Years.

yrs.	5 per cent.	6 per cent.
1	1,05000	1,06000
2	1,10250	1,12360
3	1,15762	1,19102
4	1,21550	1,26247
5	1,27628	1,33822
6	1,34009	1,41851
7	1,40710	1,50363
8	1,47745	1,59384
9	1,55132	1,68947
10	1,62889	1,79084
11	1,71032	1,89829
12	1,79585	2,01219
13	1,88564	2,13292
14	1,97993	2,26090
15	2,07892	2,39655
16	2,18287	2,54727
17	2,29201	2,69277
18	2,40661	2,85433
19	2,52695	3,02559
20	2,65329	3,20713
21	2,78596	3,39956
22	2,92526	3,60353
23	3,07152	3,81974
24	3,22509	4,04893
25	3,38635	4,29187
26	3,55567	4,74938
27	3,73345	4,82234
28	3,92012	5,11168
29	4,11613	5,41838
30	4,32194	5,74349

2. Amount of 1 $, £, &c. Annuity for any number of years, from 1 to 30.

yrs.	5 per cent.	6 per cent.
1	1,	1,
2	2,05	2,06
3	3,1525	3,1836
4	4,31012	4,37461
5	5,52563	5,63709
6	6,80191	6,97531
7	8,14200	8,39338
8	9,54910	9,89746
9	11,02656	11,49131
10	12,57789	13,18079
11	14,20678	14,97164
12	15,91712	16,86994
13	17,71298	18,88213
14	19,59863	21,01506
15	21,57856	23,27596
16	23,65749	25,67252
17	25,84036	28,21287
18	28,13238	30,90565
19	30,52900	33,75999
20	33,06595	36,78559
21	35,71925	39,99272
22	38,50521	43,39228
23	41,43047	46,99582
24	44,50199	50,81557
25	47,72709	54,85451
26	51,11345	59,15638
27	54,66912	63,70576
28	58,40258	68,52811
29	62,32271	73,63979
30	66,43884	79,05818

What is the amount of 200$ principal, and of 200$ annuity for 10 yrs, at 5 pr ct. compound interest? In Table 1, opposit 10 yrs under 5 pr ct. is 1,62889 the amt. of 1$ for that time. 1,62889 x 200=325,778$ A. amount of 200$ for 10 years.

In Table 2, *opposit 10* yrs, under 5 pr ct. is 12,57789, x200= *2515,578 A.* amouut of 200 payable yearly at 5 per cent, com*ound interest.*

3. *Preſent worth of* 1 $ *due at the end of any number of years from* 1 *to* 30.

yrs	5 pr ct.	6 pr ct
1	,95238	,94339
2	.90703	.88999
3	,86383	,88961
4	.82270	,79209
5	.78352	,74725
6	,74621	,70496
7	,71068	,66505
8	,67683	,62741
9	,64461	,59489
10	,61391	,55839
11	,58468	,56278
12	,55683	,49696
13	,53032	,46883
14	,50506	,44230
15	,48101	,41726
16	,45831	,39304
17	,43629	,37136
18	,41552	,35034
19	,39573	,33051
20	,37688	,31180
21	,35894	,29455
22	34185	,27750
23	,32557	,26179
24	,31006	,24698
25	,30530	,23299
26	,28124	,21981
27	,26784	,20736
28	,25509	,19563
29	,24294	,18455
30	,23137	,17411

4. *Preſent worth of* 1 $. *annuity for any number of years, from* 1 *to* 30.

yrs	5 pr. ct.	6 pr ct.
1	0,95238	0,94339
2	1,85941	1,83339
3	2,72325	2,67301
4	3,54595	3 4651
5	4,32948	4,21236
6	5,07569	4,91732
7	5,78637	5,58238
8	6,46321	6,20979
9	7,10782	6,80169
10	7,72173	7,36008
11	8,30641	7,88687
12	8,86325	8,38384
13	9,39357	8,85268
14	9,89864	9,29498
15	10 37966	9,71225
16	10,83777	10,10589
17	11,27407	10,47726
18	11,68958	10,8276
19	12,08532	11,15811
20	12,46221	11,46992
21	12,82115	11 76407
22	13 163	12,04158
23	13,48807	12,30338
24	13,79864	12,55035
25	14,09394	12,78335
26	14,37518	13 00316
27	14,64303	13,21053
28	14,89813	13,40616
29	15,14107	13,59072
30	15,37245	13,76483

What is the present worth of 200$ payable 10 years hence, and of 200$ payable yearly for 10 years, discounting at 5 per ct. compound interest?

In the above tables opposit 10 years, is ,61391 present worth of 1 $ due at the end of 10 years, and 7,72173 present worth of 1 payable yearly for 10 years to come. A. 122,782 present value of 200; and 1544,346 value of 200 annuity.

What annuity, being forborne 4 years, will amount to 9'6247696, at 6 per cent. compound intereſt?

Amount of 1 for 4 yrs. 4,374616)262,47696(60 A

5. *The Annuity which 1 \$ will purchaſe for any number of Years to come, from 1 to 40.*

yrs	5 pr cent.	6 pr cent.	yrs	5 pr cent.	6 pr cent.
1	1,05	1,06	21	,078	,085
2	,5378	,54544	22	,07597	,08303
3	,36721	,37411	23	,07414	,08128
4	,28201	,28859	24	,07247	,07968
5	,23097	,23739	25	,07095	,07823
6	,19702	,20336	26	,06956	.0769
7	,17282	,17913	27	,06829	,0757
8	,15473	,16103	28	,06712	,07459
9	,14 69	,14702	29	,06604	,07358
10	,1295	,13587	30	,06505	,07272
11	,12039	,12679	31	,06413	,07179
12	,11282	,11927	32	,06328	,071
13	,10645	,11696	33	,06249	,07027
14	,10102	,10758	34	,06175	,06959
15	,09624	,10296	35	,06107	,06899
16	,09227	,09895	36	,06043	,06839
17	,0887	,09544	37	,05984	,06785
18	,08555	,09235	38	,05928	,06735
19	,08274	,08962	39	,05876	,06689
20	,08024	,08718	40	,05828	,06646

What annuity, to continue 10 years, at 5 per cent compound intereſt, may be purchaſed for 200 \$. ready money?

Oppoſit 10 yrs under 5 per cent is ,1295 the annuity which 1 will purchaſe for that time. ,1295 x 200=25,90 **A.**

What annuity, to continue 25 years, will 1000 \$ purchaſe, at 6 per cent compound intereſt?

1 \$ in advance is worth a yearly payment of about 7 cts. 8,23 m. for 25 yrs. at 6 per cent. ,07823 x 1000=A.

CIRCULATING DECIMALS,

ARE thoſe conſiſting of a continual repetition of the ſame figures, ſuch as will ariſe by dividing a number by ſuch a diviſor as will not perfectly meaſure it; as 1 ÷ 3 = ,33333 &c. 34 ÷ 99 = ,34343434 &c.

The circulating figures are called *repetends*. When one figure only repeats, it is called a *ſingle repetend*; as ,1111 &c. ,444 &c. and a *compound repetend*, when two or more figures repeat, as, ,010101 &c. ,235235235 &c. The number, whoſe figures all repeat, is called a *ſimple repetend*; as, ,3434 &c. ,235235 &c. and a *mixed repetend*, if other figures ariſe before thoſe which repeat; as, $,132\dot{2}$, &c. 4,274343, &c. A ſingle repetend is repreſented by placing a point over the repeating figure; as $,\dot{1}$ repreſents ,1111 &c. $,\dot{6}=,666$ &c. A compound repetend is repreſented by placing a point over the firſt and laſt repeating figures; as, $,\dot{0}\dot{1}$ repreſents ,0101 &c. $,\dot{3}4\dot{2}=,342342342$ &c.

"*Similar repetends* begin at the ſame place; as, $,\dot{3}$ $,\dot{5}$ and $,\dot{7}$ or $\dot{3}.4\dot{2}$, & $2,\dot{1}\dot{3}$. *Diſſimilar repetends* begin at different places; as, $,\dot{4}$ $,0\dot{3}$. $\dot{7}$, &c. *Conterminous repetends* end at the ſame place; as, $,\dot{2}9\dot{5}$ $,\dot{0}4\dot{5}$ *Similar and conterminous repetends* are ſuch as begin and end at the ſame place; as, $3,4\dot{3}2\dot{1}$, $1,2\dot{7}1\dot{2}$"

REDUCTION OF CIRCULATING DECIMALS.

To reduce a ſimple repetend to its equivalent vulgar fraction.

RULE. Place 9 under each repeating place for the fraction required; but when there are integers, to be reduced, in the repetend, annex a cipher to the dividend of the fraction for every integral figure in the repetend.

EXAMPLES.

Reduce $,\dot{3}$ and $\dot{3}4\dot{1}$ to vulgar fractions.

$,\dot{3}=\frac{3}{9}=\frac{1}{3}$ A. $,\dot{3}4\dot{1}=\frac{341}{999}$ A. 999)341(,341341 &c. proof.

Reduce $\dot{1}414,1\dot{4}$. A. $1414\frac{14}{99}$ or $\frac{1414140000}{999999}$

Reduce ,0$\dot{3}\dot{2}$. $\frac{32}{9900}$ or ,00$\frac{32}{99}$ A.

Reduce 3,$\dot{3}$. 3,$\dot{3}$ = $\frac{330}{99}$ or 3$\frac{3}{9}$ A. = 3$\frac{1}{3}$.

Reduce 72,$\dot{7}\dot{2}$. A. 72$\frac{72}{99}$ or $\frac{727200}{9999}$.

Reduce ,$\dot{0}\dot{4}$. A. $\frac{4}{99}$ 99)4,00(,040404+ proof.

Reduce ,$\dot{3}8461\dot{5}$. $\frac{384615}{999999} = \frac{5}{13}$ A.

To reduce a mixed repetend to its equivalent vulgar fraction.

"Rule 1 To as many nines as there are figures in the repetend, annex as many ciphers as there are finite places, for a denominator. 2. Multiply the nines in the ſaid denominator by the finite part, and add the repeating decimal to the product, for the numerator. 3. If the repetend begin in ſome integral place, the finite value of the circulating part muſt be added to the finite part."

Or, reduce the finite and circulating parts to fractions having a common diviſor, and add them together for the fraction required.

EXAMPLES.

"What is the vulgar fraction equivalent to ,13$\dot{8}$?

9 x 13+8=125=numerator, and 900=the denominator; ,13$\dot{8}$=$\frac{125}{900}$=$\frac{5}{36}$ the anſwer"

Reduce ,15$\dot{3}$ to its equivalent vulgar fraction.

,15$\dot{3}$ = ,15 + ,00$\dot{3}$. 15 = $\frac{15}{100}$; ,00$\dot{3}$ = $\frac{3}{900}$ × $\frac{15}{100}$ = 900|00 common ÷*r*.

15 x 900 = 135|00
3 x 100 = 3|00

new ÷*nd* $\overline{138}$ $\frac{138}{900}$ A. proof 900)138(,15333+

Reduce ,4$\dot{6}$. = $\frac{4}{10}$ + $\frac{6}{90}$ (= ,0$\frac{6}{9}$, or ,0666)

$\frac{4}{10}$ × $\frac{90}{90}$ = $\frac{360}{900}$; $\frac{6}{90}$ x $\frac{10}{10}$ = $\frac{60}{900}$; + $\frac{360}{900}$ = $\frac{420}{900}$ A.

Reduce 2,$\dot{1}\dot{2}$. = $\frac{2}{1}$ + $\frac{12}{99}$ or 2$\frac{12}{99}$. $\frac{2}{1}$ x $\frac{99}{99}$ = $\frac{198}{99}$ + ($\frac{32}{99}$ x $\frac{1}{1}$ =) $\frac{12}{99}$ = $\frac{210}{99}$ A.

O

Reduce $401,100\dot{2}0\dot{1}.=401\frac{1}{10}\Big|\frac{00}{00}+\frac{201}{999000}\cdot\frac{1}{10}\times\frac{999000}{999000}$ $=\frac{999000}{9990000};+\frac{2010}{9990000}=\frac{100101}{999000}.\ 401\frac{100101}{999000}$ A.
999000)100101,(,1002012012012 &c. proof.

To make diſſimilar repetends ſimilar and conterminous.

“Rule. Change them into other repetends, which ſhall each conſiſt of ſo many places, as the leaſt common multiple of the ſums of the ſeveral numbers of places, found in all the repetends, contains units;” i.e. count the places in the ſeveral repetends, and the leaſt number, that can be divided by theſe ſeveral numbers without a remainder, is their *leaſt common multiple;* continue the diſſimilar repetends to ſo many places beyond the loweſt place among them as their laſt common multiple contains units, 1 excepted.

Examples.

Make $3,4\dot{5}$; $6,\dot{3}1\dot{7}$; $52,3$; $191,\dot{0}\dot{3}$; $,\dot{0}5\dot{7}$ ſimilar & conterminous.

In the firſt repetend there is one repeating place, in the 2d 3, in the 3d none, in the 4th two, in the 5th three.

$$\left.\begin{array}{rl} 3,4\dot{5} = & 3,45\dot{5}5555\dot{5} \\ 6,\dot{3}1\dot{7} = & 6,31\dot{7}3173\dot{1} \\ 52,3 = & 52,30\dot{0}0000\dot{0} \\ 191,\dot{0}\dot{3} = & 191,03\dot{0}3030\dot{3} \\ ,\dot{0}5\dot{7} = & ,05\dot{7}0570\dot{5} \end{array}\right\}\text{A.}$$

$3)1,\ \underline{3,}\ 0,\ 2,\ \underline{3,}$ No. of repeating figures.
$1\times\overline{1\times}\,0\times2\times\overline{1\times3}\ \div r=6$ common multiple.

Make $3,\dot{2}$ $4,\dot{3}\dot{2}$ and $4,\dot{0}\dot{4}$ ſimilar and conterminous.

$$\left.\begin{array}{lll} 3,\dot{2} & =3,\dot{2}\dot{2} \text{ or} & 3,\dot{2}22\dot{2} \\ 4,\dot{3}\dot{2} & 4,32 & 4,3232 \\ 4,04 & 4,04 & 4,0404 \end{array}\right\}\text{A.}$$

“*To find whether the decimal fraction, equal to any given vulgar one, be finite or infinite, and how many places the repetend will conſiſt of.*”

Rule. Divide the dividend by the diviſor of the fraction, until there be no *remainder*, (the fraction is then finite;)

or until the remainder be like ſome previous remainder, which will ſhow that the fraction is infinite, and the repetend in the quotient will continue from one of the ſimilar remainders to the other; (ſo any quotient may be continued without diviſion after finding two ſimilar remainders.)

1. Reduce the given fraction to its least terms, and divide the denominator by 2, 5, 10 as often as possible. 2. If the whole denominator vanish in dividing by 2, 5, or 10, the decimal will be finite, and will consist of so many places as you perform divisions. 3. If it do not so vanish, divide 9999, &c. by the former result, till nothing remain, and the number of 9's used will shew the number of places in the repetend; which will begin after so many places of figures as there were 10's, 5's, or 2's, divided by.

EXAMPLES.

Whether is the decimal equivalent to $\frac{27}{99}$ finite or infinite?

9)$\frac{27}{99}=\frac{3}{11}$. 11)3,00(,$\dot{2}\dot{7}$|272727
22
—
80
77
—
3 —*r*.

The decimal is infinite, and the second remainder being similar to the dividend shows that it is a simple repetend and consists of two places If the operation be continued, the remainders will all be similar between similar places of the quotient.

Whether is the decimal equal to $\frac{92}{448}$ finite or infinite?

4)$\frac{92}{448}=\frac{23}{112}$; 2)112=56 ÷2=28 ÷2=14 ÷2=7; therefore the decimal muſt be infinite. 7)999999
142857 As 6 nines were uſed the repetend conſiſts of 6 places, which will begin after 4 places of the quotient are found; as,

112)23(,2053$\dot{5}$71428$\dot{8}$|571428 &c.

Whether is the decimal equivalent to $\frac{16}{56}$ finite or infinite?

$\frac{16}{56}=\frac{2}{7}$. 7)2(,$\dot{2}8571\dot{4}$|$\frac{2}{7}$ It is infinite, and the repetend contains six places.

ADDITION OF CIRCULATING DECIMALS.

"RULE 1. Make the repetends ſimilar and conterminous, and find their ſum as in common addition.

2. Divide this ſum by as many nines as there are places in the repetend, and the remainder is the repetend of the ſum; *which muſt* be ſet under the figures added, with cipher *on the left*, when it has not ſo many places as the repeten

3. Add the quotient of this diviſion to the next column, and proceed with the reſt as in finite decimals."

Or, add the whole after they are made ſimilar and conterminous, as in ſimple addition, and add the ſame, to the right hand figure of the ſum, that was added to the firſt column at the left of the repetends; (which is the number of tens in the left repeating column, and is the ſame that would be added if each decimal were infinitely extended.)

EXAMPLES.

Add $3,\dot{6}$, $78,3\dot{4}7\dot{6}$, $735,\dot{3}$ and $562,\dot{6}\dot{7}$ together.

$3,\dot{6} = 3,\dot{6}66666\dot{6}$
$78,3\dot{4}7\dot{6} = 78,3\dot{4}76476$
$735,\dot{3} = 735,\dot{3}33333\dot{3}$
$562,\dot{6}\dot{7} = 562,\dot{6}767676$

A. $1380,0\dot{2}4415\dot{3}$

Here the ſum of the repetends is $2\dot{2}4415\dot{1}$, the 2 at the left being added to 1 at the right makes the right hand figure of the repetend 3, or if the ſum be ÷*d* by 999999 the qt. will be 2 & the remainder (244153) the repetend.

Add $9,\dot{8}1\dot{4}$ $1,5$ $87,2\dot{6}$ $,08\dot{3}$ and $124,\dot{0}\dot{9}$ together.
A. $222,75\dot{5}7239\dot{0}$.

Add together $,\dot{1}6\dot{2}$ $134,\dot{0}\dot{9}$ $2,\dot{9}\dot{3}$ $97,2\dot{6}$ $3,\dot{7}6923\dot{0}$
$99,08\dot{3}$ $1,5$ $,\dot{8}1\dot{4}$ A. $339,62\dot{6}5107\dot{7}$

SUBTRACTION OF CIRCULATING DECIMALS.

RULE. Make the repetends ſimilar and conterminous, & proceed as in ſimple ſubtraction, obſerving to diminiſh the right hand figure of the remainder 1, when the repetend of the ſubtrahend exceeds that of the minuend.

From $85,\dot{6}\dot{2},= \quad 85,6\dot{2}62\dot{6}$
Subtract $13,\dot{7}64\dot{3}2= \quad 13,7\dot{6}43\dot{2}$
Difference $= \quad 71,8\dot{6}19\dot{3}$

From $476,3\dot{2}$ take $84,767\dot{7}$. A. $391,55\dot{4}$

From $3,85\dot{6}\dot{4}$ take $,03\dot{8}\dot{2}$. A. $3,81818181$ &c.

MULTIPLICATION OF CIRCULATING DECIMALS.

Rule. Reduce the given repetends to their equivalent vulgar fractions, and proceed as in multiplication of fractions for the product; which may be reduced to its equivalent decimal, as in vulgar fractions.

Multiply $,\dot{3}\dot{6}$ by $,2\dot{5}$. $,\dot{3}\dot{6}=\frac{36}{99}=\frac{4}{11}$. $,2\dot{5}=\frac{23}{90}$.
$\frac{4}{11}\times\frac{23}{90}=\frac{92}{990}$ A. $=,0\dot{9}2\dot{9}$

Multiply $87,2\dot{3}$ by $,2\dot{6}$. A. $9,92\dot{8}$

Multiply $8574,\dot{3}$ by $87,\dot{5}$ A. $750730,\dot{5}1\dot{8}$.

DIVISION OF CIRCULATING DECIMALS.

Rule. Reduce the given repetends to their equivalent vulgar fractions, and proceed as in diviſion of fractions for the quotient; which may be reduced to its equivalent decimal if neceſſary.

Divide $,\dot{3}\dot{6}$ by $,2\dot{5}$. $,\dot{3}\dot{6}=\frac{36}{99}=\frac{4}{11}$ $,2\dot{5}=\frac{23}{90}$.
$\frac{4}{11}\div\frac{23}{90}=\frac{4}{11}\times\frac{90}{23}=\frac{360}{253}$ A. $=1\frac{107}{253}=1,4\dot{2}2924901185770750988\dot{1}$.

Divide $319,28007\dot{1}1\dot{2}$ by $764,5$. A. $,4176\dot{3}2\dot{5}$

Divide $234,\dot{6}$ by $,\dot{7}$. A. $301,\dot{7}1428\dot{5}$

"PRACTICE

"IS a contraction of the Rule of Three direct, when the firſt erm happens to be an unit, or one, and has its name from ts frequent uſe in buſineſs.

"Case 1. *When the price is an aliquot, or even part of a ſhilling.*

Rule. Divide the given number by the part, and the *uotient is the anſwer* in ſhillings; what remains is to be re *uced as in* Compound Diviſion.

THE TABLE.

Parts of a £.

s.	d.		
10		is	$\frac{1}{2}$
6	8	. . .	$\frac{1}{3}$
5		. . . .	$\frac{1}{4}$
4		. . . .	$\frac{1}{5}$
3	4	. . . .	$\frac{1}{6}$
2	6	. . . .	$\frac{1}{8}$
2		. . . .	$\frac{1}{10}$
1	8	. . . .	$\frac{1}{12}$
1		. . . .	$\frac{1}{20}$

Parts of a shilling.

d.		
6	is	$\frac{1}{2}$
4	. . . . .	$\frac{1}{3}$
3	. . . . .	$\frac{1}{4}$
2	. . . . .	$\frac{1}{6}$
$1\frac{1}{2}$	. . . . .	$\frac{1}{8}$
1	. . . . .	$\frac{1}{12}$

Parts of a Tun.

Cwt.	*Qr.*		
10		is	$\frac{1}{2}$
5		. . . .	$\frac{1}{4}$
4		. . . .	$\frac{1}{5}$
2	2	. . . .	$\frac{1}{8}$
2		. . . .	$\frac{1}{10}$
1		. . . .	$\frac{1}{20}$

Parts of a Cwt.

Qrs.	*lb.*		
2		is	$\frac{1}{2}$
1		. . . .	$\frac{1}{4}$
	16	. . . .	$\frac{1}{7}$
	14	. . . .	$\frac{1}{8}$
	8	. . . .	$\frac{1}{14}$
	7	. . . .	$\frac{1}{16}$
	4	. . . .	$\frac{1}{28}$
	2	. . . .	$\frac{1}{56}$

Parts of $\frac{1}{2}$ Cwt.

lb.		
28	is	$\frac{1}{2}$
14	. . . . . .	$\frac{1}{4}$
8	. . . . . .	$\frac{1}{7}$
7	. . . . . .	$\frac{1}{8}$
4	. . . . . .	$\frac{1}{14}$
$3\frac{1}{2}$	. . . . . .	$\frac{1}{16}$
2	. . . . . .	$\frac{1}{28}$

Parts of a $\frac{1}{4}$ Cwt.

lb.		
14	is	$\frac{1}{2}$
7	. . . . . .	$\frac{1}{4}$
4	. . . . . .	$\frac{1}{7}$
$3\frac{1}{2}$	. . . . . .	$\frac{1}{8}$
2	. . . . . .	$\frac{1}{14}$
1	. . . . . .	$\frac{1}{28}$

What will 4596 yards coſt at 6 d. per yard ?

6 d. = $\frac{1}{2}$ s. | 2)4596(2298 s. ÷ 20 = 114 £. 18 s. A.

or $\overline{12\text{d} \times 20\text{s}}$ ÷ 6 = 40)4596(114 £. 18 s. A.

3746 *Yards*, at 4 d. per yard. - - - A. 62 £. 8 s. 8 d.
1095 3 - - - - - - - - - - 13 13 9
7596 2 - - - - - - - - - - 63 6 0

" CASE 2. *When the price is pence, or pence and farthings, and no even part of a ſhilling.*

RULE. Find the even parts for the price, and proceed as in Caſe 1, and the ſum of the quotients is the anſwer.

What will 4937 yards come to, at 9 d. per yard ?

6	$\frac{1}{2}$	4937
3	$\frac{1}{2}$	2468 .6
		1234 3
2,0		370|2 9 = 185 £. 2 s. 9 d. A.

or $\overline{4937} \div \overline{\overline{12\text{d} \times 20\text{s}}} \times 9\text{d} = A.$

or $\frac{\ }{12} \times \frac{9}{20} = \frac{3}{80} \times 4937 = A.$

2765 *Yards*, at	8 d. per yard. - - - -	A. 92 £.	3 s.	4 d.
3762	7 - - - - - - - - -	109	14	6
3159	7½ - - - - - - - - -	98	14	4½
2563½	9½ - - - - - - - - - -	101	9	5¼

"Case 3. *When the price is ſhillings, or ſhillings and pence, and an even part of a pound.*

Rule. Divide the given quantity by the even part, and the quotient is the anſwer in pounds. If there be a remainder, reduce it as in Compound Diviſion.

At 6 s. 8 d. per yard, what will 473 yards come to ?

6 s. 8 d. = $\frac{1}{3}$ £. 3)473(157 £. 13 s. 4 d. A.

or 6 s. 8 d. = $_{12} \times \frac{80}{20} = \frac{1}{3}$ £. x 473 = A.

387 *Yards* at	10 s. per yard. - - -	A. 193 £.	10 s.	0 d.
397	3 4 d. - - - - - - -	66	3	4
797½	2 6 - - - - - - - -	99	13	9

"Case 4. *When the price is ſhillings or ſhillings and pence, which make no even part of a pound.*

Rule. Find the even parts for the price, and divide as in Caſe 3, or multiply the given quantity by the ſhillings, and take the even parts of ſhillings for the pence, as in Caſe 2.

What coſt 287 yards at 17 s. 6 d. per yard ?

First method.

s. *d.*		287
10	$\frac{1}{2}$	143 10
5	$\frac{1}{2}$	71 15
2 6	$\frac{1}{2}$	35 17 6

A. 251*l.* 2s. 6 d.

Second method,

287

17 6

2009

287

6 | $\frac{1}{2}$ | 143 6

2|0)502|2 6

251*l.* 2s. 6 d.

or 17[illegible] 6 d. = $_{12} \times \frac{210}{20} = \frac{7}{8}$ £. x 287 = A[illegible] = 251*l.* 2s. 6 d.

8172 *Yards* at	15 s. - - - - - -	A. 6129 £.		
4765	11 8 d. - - - - - -	2779	11 s.	8 d
96½	2 9½ - - - - - - -	13	9	4

"CASE 5. *When the price is an even number of ſhillings.*

RULE. Multiply the quantity by half the ſhillings, doubling the firſt (or right hand) figure of the product for ſhillings, the reſt are pounds.

What will 788 yards come to, at 2 ſhillings per yard?

788
1 = half the ſhillings.

£. 78 16 Ans. or 2 s. = $\frac{1}{10}$ £. x 788 = A.

347 *Yards.*	at 4 s. - - - - - - - -	A. 69 £. 8 s.
638	6 - - - - - - - - -	191 8
589¼	8 - - - - - - - - - -	235 14

"CASE 6. *When the price is pounds, ſhillings, &c.*

RULE. Multiply the integers of the given quantity by the pounds, and work for the ſhillings, &c. by ſuch of the preceding rules as you think beſt, and work likewiſe for the fractional parts of the integer; the ſum of theſe will give the anſwer.

What will 173 cwt. 1 qr. 14 lb. of ſugar come to, at 3 £. 15 s. 6 d. per cwt.?

		173	1	14
		3	15	6
s. d.		519		
10	½	86	10	
5	½	43	5	
6	$\frac{1}{10}$	4	6	6
1 qr.	¼	0	18	10½
14 lb.	½	0	9	5¼
	A. £.	654	9	9¾

or 173 cwt 1 qr. 14 lb. = 173,375 cwt.
3 £. 15 s. 6 d = 3,775 £.

654,490625 £. A.
20

s. 9,812500

cwt.	qrs.	lb.		s.	d.		£.	s.	d.
219	2	19	at	69	11	A	767	19	6¼
310	3	22	- - -	53	8	- - - -	834	7	5¼

In working queſtions of this kind, when the quantity is not above the multiplication table, the following method is used

What will 45 cwt. 2 qrs. 14 lb. of ſugar come to, at 3 £. 7 9 per cwt.?

3 £. 7 s. 9 d. x 5 = 16 18 9 x 9 = 152 8 9 price of 45 cwt.
2 qr. ½ 1 13 10½ price of 2 qr.
14 lb. ¼ 0 8 5½ price of 14 lb.

154 11 1

57cwt.	*2qrs.*	*8lb.*	-	3£	17s.	9d.	-	223£.	16s.	2d.
19	3	13	- -	2	5	10	- - -	45	10	6
75	3	25	- -		48	5	- - -	183	18	4¼

What will 37 cwt. 3 qrs. 7 lb. of ſugar come to, at 14 $. 40 cts. per cwt. ?

		14 40
		37
		10080
		4320
2 qrs.	½	720
1 qr.	½	360
7 lb.	¼	90
A. $.		544,50

or 37 cwt. 3 qr. 7 lb.= 37,8125 cwt.
14,4
A. $ 544,50

tun	cwt	qr.	lb.		dls.	cts.		dls.	cts.	
24	18	3	18	of hemp at	289	50	pr tun.	7221	73	A
31	16				268	75		8546	25	
19	14	2	12	iron	110			2170	33	8
	17	3	24	cordage	14		per cwt.	251	50	

What will 8767 feet of merchantable boards come to, at 38 s. 6 d. per M. ? A. 16£. 17 s 6 d.

What is the amount of a ſeaman's wages from the 15th of March to the 6th of December following, being 8 months and 20 days, at 16 dollars per month ? A. 138,66⅔ $.

At 4£. 11 3 per cwt. what will 3 qrs. 25½ lb. come to ?
A. 4£. 9 s. $2\frac{125}{224}$ d.

What will 19 cwt. 3 qrs. 27½ lb. come to, at 19£. 19 s. 11¾ d. per tun ? A. 399£. 19 s. $5,\frac{10641}{17920}$.

MISCELLANEOUS APPLICATION OF RULES.

"RULE established by the Courts of Law in Massachusetts for making up judgments on SECURITIES FOR MONEY, which are upon Interest, and on which partial payments have been endorsed.

COMPUTE the interest on the principal sum, from the time when the interest commenced to the first time when a payment was made, which exceeds either alone or in conjunction with the preceding payments (if any) the interest at that time due : add that interest to the principal, and from the sum subtract the payment made at that time, together with the preceding payments (if any) and the remainder forms a new principal ; on which compute and subtract the interest, as upon the first principal : and proceed in this manner to the time of the judgment. By this Rule, *the payments are first applied to keep down the interest ; and no part of the interest ever forms a part of a principal carrying interest.*

A, by his note dated January 1, 1780, promises to pay B 1000 dollars in six months, with interest from the date.

On his note are the following endorsements:

Received, April 1, 1780, 24 dols.—August 1, 1780, 4 dols.—Dec. 1, 1780, 6 dols.—Feb. 1, 1781, 60 dols.—July 1, 1781, 40 dols.—June 1, 1784, 300 dols.—Sept. 1, 1784, 12 dols.—Jan. 1, 1785, 15 dols.—and Oct. 1, 1785, 50 dols.—& the judgment is to be entered Dec. 1, 1790."

		$. cts.
The principal sum carrying interest from January 1, 1780,		1000 00
Interest to April 1, 1780, (3 months) - - - - - - - - - -		15 00
Amount		1015 00
Paid April 1, 1780, a sum exceeding the interest		24 00
Remainder for a new principal		991 00
Interest on 991 $ from April 1, 1780, to Feb. 1, 1781, (10 mo.)		49 55
Amount		1040 55
Paid August 1, 1780, a sum less than the interest then due	$	4 00
Paid Dec. 1, 1780, do. - - - do. - -		6 00
Paid Feb. 1, 1781, do. greater than the interest then due		60 00
Remainder for a new principal		970 55
Interest on 970,55 D from Feb. 1, 1781, to July 1, 1781 (5 mo.)		24 26
Amount		994 81
Paid July 1, 1781, a sum exceeding the interest - - -		40 00
Remainder for a new principal		954 81
Interest on 954,81 D from July 1, 1781, to June 1, 1784, (2 years 11 months)		167 09
Amount		1121 90
Paid June 1, 1784, a sum exceeding the interest - - - -		300 00
Remainder for a new principal		821 90
Interest on 821,90 D from June 1, 1784, to Oct. 1, 1785, (1 year 4 months)		65 75
Amount		887 65
Paid Sept. 1, 1784, a sum less than interest then due	D12 00	77 00
Paid Jan. 1, 1785, do. - do. - - - -	15 00	
Paid Oct. 1, 1785, do. greater with two last payments than interest then due - - -	50 00	
Remainder for a new principal		810 65
Interest on 810,65 D from Oct. 1, 1785, to Dec. 1, 1790, the time when judgment is to be entered (5 yrs 2 m.)		251 30
Judgment rendered for the Amount - - - - -		1061 95

"BUYING AND SELLING STOCKS.

STOCK, in the sense in which it is here used, is a fund established by government or individuals in a corporate capacity, the value of which is variable.

What is the amount of 1565 dollars national bank stock, at 134 per *cent?*
1565 x 1,34=2097,1 D A.

What is the amount of 2958 dolls. bank stock at 25 per cent. advance?
2958 x ,25=739,5,+2958=3697,5 A.

To find a fraction in its shortest terms nearly equivalent to any given fraction.

Divide the divisor of the given fraction by its dividend, and the dividend by the remainder, the first remainder by the second, &c. place 1 over the first quotient, which will be the nearest fraction having 1 for its dividend; multiply both parts of this fraction by the second quotient, and add one to the product of the divisor, for the second nearest fraction, which multiply by the third quotient as before, and add the first found fraction, &c. till a fraction, sufficiently near to the given, be obtained.

Reduce ,785398 (which is near the area of a circle whoſe circumſcribing ſquare is 1,) to a vulgar fraction nearly equivalent.

,785398$=\frac{785398}{1000000}$, 785398)1000000(1 1, 3, 1, 1, 1, 15, qts.
$\frac{1}{1}$, firſt ratio, or fr. 785398
$\frac{1}{1}\times\frac{3}{3}=\frac{3+0}{3+1}=\frac{3}{4}$, 2d ratio
214602(785398(3
$\frac{3}{4}\times\frac{1+}{1+}\frac{1}{1}=\frac{4}{5}$, 3d ratio 643806
$\frac{4}{5}\times\frac{1}{1}$ (4th qt.)$\genfrac{}{}{0pt}{}{+}{+}\frac{3}{4}=\frac{7}{9}$, 4th ratio 141592)214602(1
$\frac{7}{9}\times\frac{1}{1}$ (5th qt.)$\genfrac{}{}{0pt}{}{+}{+}\frac{4}{5}$ (3d ratio)$=\frac{11}{14}$, 5th ra. (commonly uſed.)
$\frac{11}{14}\times\frac{15+}{15+}\frac{7}{9}=\frac{172}{219}$ 6th ratio.

As 1 : $\frac{11}{14}$+ :: the ſquare of the diameter : the area of a circle.

The circumference of a circle is to its diameter as 3,1416 is to 1 nearly; what vulgar fraction will expreſs the proportion ? $3\frac{1}{7}=\frac{22}{7}$ or $\frac{355}{113}$ A. nearly.
3,1416$=3\frac{1416}{10000}$ 1416)10000(7 7, 16, 11 qts. $\frac{1}{7}$, 1ſt ratio.

What is the difference between twice five and twenty, and twice twenty five ? A. 20

A was 2 years old when B was 23 ; how old was A when B was 47 ; and B when A was 60 ?
A's age 26, and B's 81. A.

What number, taken from the ſquare of 48, will leave 16 times 54 ? A. 1440.

What number, added to 1 of 3813, will make a ſum *equal to 200 ?* A. 77.

Divide 1000 $ between A B and C, ſo that A may have 120 more & B 95 leſs than C. $\overline{95-120\ 1000}\div 3=$ C's ſhare, C's $-95=$ B's ſhare, B's $+25+95=$ A's ſhare.

A father dividing his fortune among his ſons, gave A 4 $ as often as B 3, and C 5 as often as B 6; what was the whole legacy, ſuppoſing A's ſhare 5000 $?
As 8 ſhares : 5000 :: 8 + 6 + 5 ſhares : the amount.

What number is that to which if $\frac{2}{7}$ of $\frac{5}{9}$ be added, the ſum will be 1? $\frac{53}{63}$ A.

A young man ſpent $\frac{1}{4}$ of his fortune in 8 months, $\frac{3}{7}$ of the remainder in 12 months, then having 410 £ left; what did he ſpend per month? A. 27 £ 6 s 8 d.

" A farmer being aſked how many ſheep he had, anſwered, that he had them in 5 fields; in the firſt he had $\frac{1}{4}$ of his flock, in the ſecond $\frac{1}{6}$, in the third $\frac{1}{8}$, in the fourth $\frac{1}{12}$, and in the fifth 450; how many had he?" A.
$\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{12}=\frac{15}{24}$, $-\frac{24}{24}=\frac{9}{24}$ of flock $=450$, $\div\frac{9}{24}=1200$

What is the difference between the intereſt and diſcount of 350 £, at 4 per cent for 8 years? A. 27 £ 3+ s.

If a tax of 3000 $ be laid on a town, and the inventory of all the eſtates in the town amount to 40000 $, what muſt A pay, whoſe eſtate is 500? 37,5 $.

If the earth revolve 366 times in 365 days, in what time does it perform one revolution?
As 366 rev. : 365 days :: 1 rev. : 23 h. 56′ 3″ 56‴+ = 1 ſiderial day, or the ſpace of time which happens between the departure of a Star from, and its return to the ſame meridian again.

If the earth make one complete revolution in 23 h. 56′ 3″ in what time does it move one degree? 3′ 59″ 20‴ A.

If the earth perform its diurnal revolution in 24 hours; in what time does it move one degree?
A. 4 minutes $=\frac{1}{360}$ of a ſolar day, or the ſpace of time which intervenes between the Sun's departure from any meridian and its return to the ſame ag

What length of board $8\frac{3}{8}$ in. broad is equal in area to a ſquare ft. ? A. $17'\frac{1}{7}$ in.

If the time paſt ſince noon be equal to $\frac{4}{5}$ of the time till midnight; what time in the day is it ?

A. 20 min. paſt 5 o'clock.

Seven eights of what number is 6 more than $\frac{4}{5}$ of the ſame ?

A. 80

Hiero, king of Syracuſe, gave orders for a crown to be made of pure gold ; but ſuſpecting the workmen had debaſed it, he recommended the diſcovery of the fraud to the famous Archimedes ; who in order to detect the impoſition, procured a maſs of pure gold, and another of ſilver or copper, each equal in weight to the crown ; and each, being put ſeparately into a veſſel full of water, expelled a certain quantity, which determined their ſpecific bulks.

If the weight of each were 10 lb. and the water expelled by the alloy ,92 lb. by the gold ,52 lb. and by the compound crown ,64 lb. what were the quantities of gold and alloy in the crown ?

Simples ,925 2, –,64 mixture. A. ,12 lb of alloy to ,28 of gold or (x 25=) 3 parts alloy to 7 gold.

To what diſtance would the lower end, of a bar of iron or wire 3 ft long, ſink in a fountain of quickſilver ; if the bar, being equal in all places, were kept perpendicularly and its ſpecific gravity 7,645 and that of the quickſilver 13,6 ?

As gravity of the quickſilver : that of the iron :: 3 : A. 1,686+ ft.

If a chaiſe run 6 miles an hour, how much faſter does the top of the wheel move, than the bottom, or than that part which is 1 ft from the ground, the wheel being 5 ft high ; and what length of road would it go while it turned a wheel, having 100 cogs, 20 times round, by moving it 5 cogs at each revolution (of the carriage wheel) ?

As 2,5 ft heit. of axle : 6 velocity of axle :: 5 ft. top : 12 mi. an hour vel. of top (–0 vel. of bottom=12.) A =5 times as faſt as that 1 ft from bottom ; and 5 ft x 3,1416 x 20 x $\overline{100 \div 5}$ =whole diſtance.

If A can do a piece of work in 10 days and B the ſame in 13 days; in what time can both working together finiſh it? both can do 10+13 pieces in 10 x 13 days. A. $5\frac{15}{23}$ d.

What is the length of a pendulum, which will vibrate once in 3 ſeconds, and of one which will vibrate in $\frac{1}{2}$ ſecond?
$3 \times 3 \times 39{,}2 =$ A. $,5 \times ,5 \times 39{,}2 =$ A. $= \frac{1}{2}$ ſec. pendulum.

Five perſons depart from the ſame place at the ſame time; A travels 35 miles, B 30 eaſt each day, C 30 ſouth, D 30 weſt, and E 30 north; how far are they from each other at the end of 7 days? A. A is 7 times 5 mi. from B, 35 x 7 and 30 times 7 from D, and $\sqrt[2]{35 \times 7 + 30 \times 7}$ from C and from E, B is 30 times 7 + 30 times 7 from D, and the ſquare root of that diſtance from C and E, &c.

If a bullet in $1\frac{3}{4}$ ſecond fall from the top of a tree; what is its height? $1\frac{3}{4} = 7$ qr. $\times 7 = 49$ ft. A.

"If 12 oxen eat up $3\frac{1}{3}$ acres of graſs in 4 weeks, and 21 oxen eat up 10 acres in 9 weeks; how many oxen will eat up 24 acres in 18 weeks, the graſs to be growing uniformly?
36 oxen. A.

The ſquare root of the area of a circle is $\frac{31}{110}$ of the circumference, or $\frac{31}{35}$ of the diameter, nearly; ſo if the ſquare root of the area be divided by $\frac{31}{110}$ or ,282 the qt. is the circumference, or by $\frac{31}{35}$ or ,886 the qt. is the diameter; or $\times$, the circumference by $\frac{31}{110}$ for the ſquare root of the area, &c.

What diſtance may the points of a pair of compaſſes be aſunder to form a circle containing a ſquare foot or 144 in. and what diſtance muſt one point move to form ſuch circle?
A. 6,774+ extent of the points $= \frac{1}{2}$ di. and 42,58+ cir.

What length of rope may be tied to a horſes head and the other end to a ſtake, to give him liberty of eating an acre of graſs? 7,14+ rods. A.

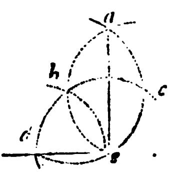
GEOMETRY,

IS the ſcience of quantity, extention or magnitude ; and may be conſidered as the art of reducing unknown to known quantities,* and of comparing different lengths, ſuperficies, and ſolids together.

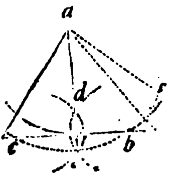
GEOMETRICAL PROBLEMS.

To form a right angle or erect a perpendicular line.

EXAMPLES.

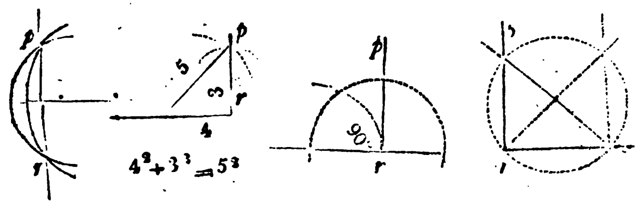


Set one foot of the compaſſes or dividers in a given point ; as, *e*, form an arch *c b d* ; then with one foot in *d*, bring the other to *b*, form an arch *a c*, and with one foot in *c* croſs the arch at *a*, and a line from *a* to *e* is the *perpendicular*, which forms the *right angle* at *e*.



Or with convenient radius, from any (given) point above the baſe, as *a*, for centre, form an arch croſſing the baſe, *b c* ; from centre *c*, form an arch at *e* or *d*, and from centre *b*, croſs the arch at *e* or *d*, and draw a line from *a*, as before for the perpendicular required.

Perpendiculars differently erected.



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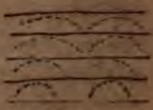
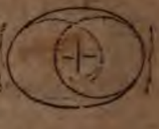
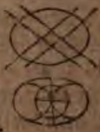
* Any thing whatever is its own perfect measure, yet its contents can not be comprehended or comparatively valued until it be reduced to some known quantity ; as inches, feet, miles, acrs, gallons, pounds, &c. each of which has by use acquired a definite meaning ; and though we may have a more accurate conception of the size of a thing by sight, *than by hearing of its* contents in such known quantities ; yet, its value *weight, &c cannot* be so accurately computed, as by comparison to *that whose value*, weight, &c. is known.



To deſcribe parallel lines. Form an arch of the ſame ſize at each end of a baſe, and draw a line from the top of one arch to that of another, for the parallel required.

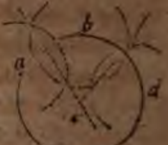


To deſcribe an ellipſe or oval. Form a ſmall circle and ſet one foot of the compaſſes in one ſide of the circle, with the other form the oppoſit ſide of the oval; ſet one foot in the part neareſt to the ends of the oval to form the ends.

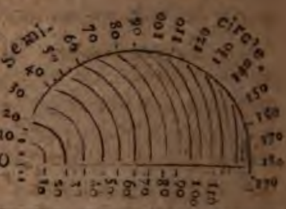


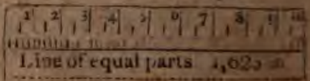
Or draw parallel lines acroſs each other and deſcribe arches as before from the points of meeting till they interſect.

Or carry a ſtring with the ends tied together round two fixed points; as *a b*.



To deſcribe a circle which will paſs through any three points, (as *a b d*) *not in a ſtraight line.* Set one foot of the dividers in each point and form an arch ſo that the arches croſſed will point to the *centre* of the *circle* required; as, *c*. In the ſame manner the centre of any given circle may be found.

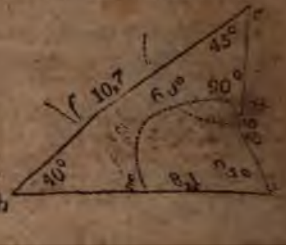




To meaſure or form angles, by the diviſions of a circle; as in T. 14.

Extend the dividers 60° on any circle, or line of chords, as from 0 to ^{60}; ſet one foot in the given point, as *b*, with the other form an arch *g f*, and take the diſtance between the lines on the arch and apply it to the circle or line of chords, which will ſhow the number of degrees the angle contains, as *b* 40°. Subtract two angles of a triangle from 180° *and the remainder is the third angle*; as $\overline{40+95}-180=45^\circ$ *e*.

Any angle may be formed y taking the given number



of degrees; as 95° *i* in the dividers and ſetting one foot in the arch or ſweep of 60 as at *g*, then the other foot will reach to the place where the line *e i* ſhould be drawn from the given point *i* to form the angle required, 95°.

Rright lines are meaſured by the equal diviſions of a right line, and circular or curve lines by equal parts of a circle or by (the unequal parts of a right line) a line of chords, as from 0 to 180 in the preceeding ſemicircle.

TRIGONOMETRY,

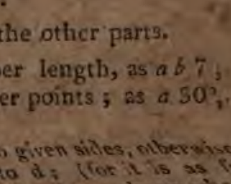
By Geometrical Conſtruction,

Is the art of protracting and meaſuring, with ſuitable inſtruments the ſides, angles, &c. of plane triangles.

Three parts of a triangle muſt be given; viz. the three ſides; two ſides and one angle, or one ſide and two angles to find the parts required. In a right triangle one angle is always given; viz. 90°

Two ſides and an angle* between them given, to find the other ſide, angles, perpendicular and contents.

Rule. Draw one of the given ſides; as *a b* 6,5, then take the length of the other given ſide in the dividers, as *b c* 3,75, and place one foot in the given point, *b*, with the other form an arch, *c d*, lay out the given angle; as *b* 80° 30′, and draw the line to the arch *c d* from thence to the point *a* 7, is the ſide required; the ſhorteſt diſtance from the point *c* to the baſe is the perpendicular *c e*, 3,7; meaſure the angles *a c* as before directed. The area may be found as under multiplication by multiplying the baſe and half the perpendicular together.



½ per.
2)3,7(1,85 x 6,5= 12,025 area.

One ſide and two angles given to find the other parts.

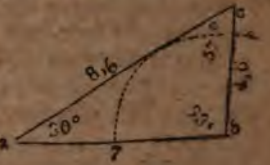
Rule. Draw the given ſide its proper length, as *a b* 7; then lay out the angles from their proper points; as *a* 30°,

---

* The angle given ſhould be between the two given ſides, otherwiſe *there may be a* miſtake made, as from e to d; (for it is as far *from b to one as to the other*,) so we ſhould be at a loss to know whether *the ſide a c were* 4,2 or 7, except the place of its touching the ar *were determined*, as at d or c.

P 2

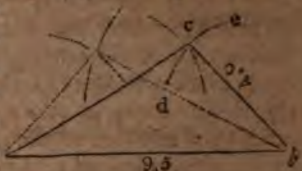
b 93° and draw lines till they meet in the other point and form the angle required; the length of theſe lines are the ſides required. The angle at *b*, being larger than a right angle obliges the oppoſit ſide to be uſed as a baſe and the ſhorteſt diſtance from *b* to the oppoſit ſide is the perpendicular 3,7. Or lay out as many degrees from the given point *b* as the oppoſit angle *a* or *c* is leſs than 90° for the perpendicular required.



The three ſides given, to find the angles, &c.

RULE. Take the length of one of the given lines; as *a* 9,5 to *b* draw a line of this length; *a b*, then take the length of another given ſide; *b* 4,6 *c*, in the dividers, ſet one foot in its proper point *b* with the other form an arch *d e*; take the length of the other line *c* 7,2 *a* ſet one foot in *a* with the other croſs the arch or curve *d c*, and draw lines from *c* to *a* and *b*. The angles may be meaſured and the contents found as before.

a 9,5 to *b*
b 4,6 *c*
c 7,2 *a*



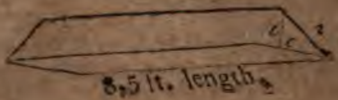
EXAMPLES.

What is the contents of a ſtick of timber 8,5 ft long, whoſe ends are equal, and each of the following dimenſions, viz. one ſide *a* 2,5 ft. *b* 1,8 ft and *c* 2 ft.?

here 3 ſides are given,

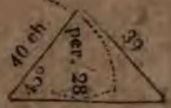
viz. *a* 2,5= ——
b 1,8= ——
c 2, = ——
their per. 1,4= ——

2,5 baſe
,7 ½ per.
———
1,75 area
8,5 length
———
14,875 ſolid. A.



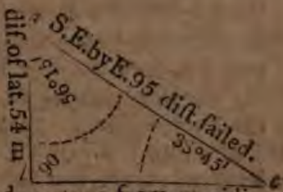
What is the area of a piece of land, two ſides being meaſured viz. 40 Chains and 54 Ch. the angle between the ſides 45° ?

54 Ch. baſe
14 — $=\frac{1}{2}$ per.
——
756 Ch. A.=75,6 acres



If a ſhip leaving a port ſail S. E. by E. (i. e. S 5 points or 56°15′ eaſt) 95 miles ; what would be her difference of latitude and departure, (i. e. how far ſouth and how far eaſt would ſhe be from the port ?)

Here two angles are given, viz. 56° 15′ and 90°; for every meridian or degree of longitude cuts each degree or parallel of latitude at right angles, as at *b*. 56° 15 — 90=33° 45′ angle required at *c* from thence a line to *b* is



departure from meridian
of port 80 miles }
dif. of lat. 54 } A.

To meaſure, and delineate the ſides, angles, &c. of any given plane.

Rule. Take the courſes with a compaſs ſuited to that purpoſe, and meaſure the diſtances on each courſe ; then on a fixed point ; as, *a* place the centre of a circle,* ſemicircle or ſomething that ſhall repreſent the compaſs, (or horizon ;) and point off the firſt courſe by the circle as it was taken with the compaſs, and meaſure the diſtance by a ſcale of equal parts, place a point at the end as *b* ; lay a ſcale of equal parts at this point parallel to the ſecond courſe on the circle, meaſure and mark the ſecond diſtance, from this point *c* lay the ſcale parallel to the third courſe, meaſure and proceed as before through the whole.

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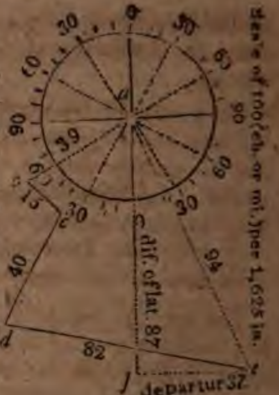
* *When a circle of* brass, paper, &c. is used as above (it should remain fixed till all the* courses are taken.

EXAMPLES.

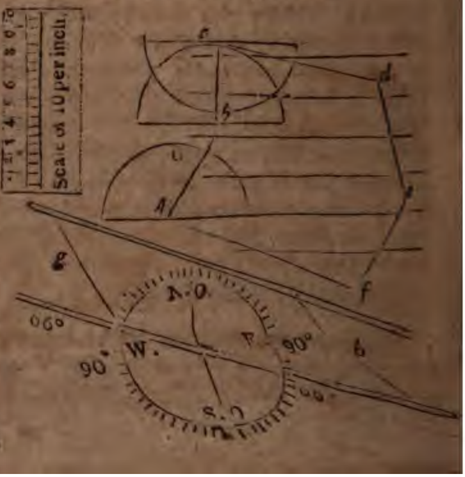
From *a* S 60° W 39 Ch. to *b*.
from *b* S 45° E 15 — to *c*.
from *c* S 30° W 40 — to *d*.
from *d* S 71° E 82 — to *e*.
from *e* N 23° W 94 — to *a*.



Here the courſes and diſtances are given, as from *a* the centre of the circle (repreſenting the compaſs) S. 60° Eaſt the diſt. is 39 Ch. to *b*. the next courſe being out of the circle, lay the ſide of a parallel in the given courſe 45° extend the other ſide to *b* and draw the line 15 to *c*; or lay a ruler parallel to the given courſes and draw the lines their proper lengths; as *b c* 15, *c d* 40, &c.

If a ſhip had run all the courſes and diſtances (in miles or leagues) except the laſt in the preceding example, then *e* would repreſent the ſhips place and the ſeveral lines the path in which ſhe run, and the right line from *a* (in the centre repreſenting the place ſailed from) to *e*, her true courſe and diſtance gained, and the N and S line or meridian *a f* her difference of latitude, and the E and W line *e f*, her departure from the meridian; (ſuppoſing the earth a plane.)

From a field book. *A*, the place of beginning, Thence
N bearing E 29 20′ 10 Ch. 55 *l*. to *b*.
N — E 10° 8 30 - to *c*
S — E 83° 17 to *d*
S — E 10° 13 20 - to *e*
S — W 34° 10 20 - to *f*
N — W 66° 20 80 - to *A*



Here the parallel lines repreſent eaſt and weſt, the ſemicircle, the compaſs and the lines *A*, *b*, *c*, *d*, *e*, *f*, the ſides of the land.

Every line in any chart or plat will lie parallel to a line of the ſame bearing in any circle or ſemicircle whoſe meridian is parallel to that in the chart; ſo by the help of a parallel, (as *g h* one ſide of which lies in the laſt courſe 66° and the oppoſit ſide near the laſt line *A f*) the courſes may be more eaſily and accurately marked out without moving the circle than in any other way.

The area of any plane may be aſcertained by reſolving it into triangles; as by drawing lines from *e* to *a*, *b*, and *c*, the above figure may be reſolved into four triangles, two of which may have one baſe.

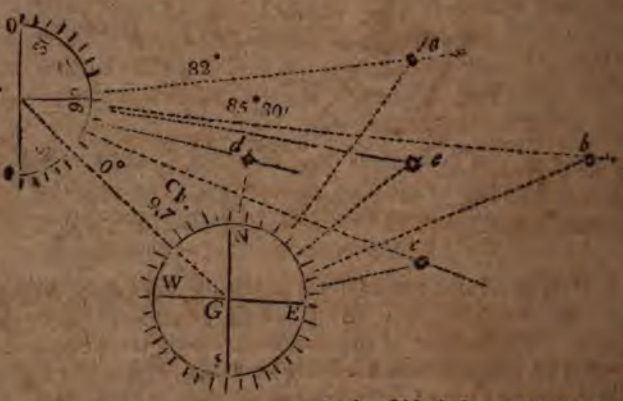
To meaſure the bearings, heights and diſtance of objects, and to delineate them on a plane.

Rule. From a fixed ſtation; as *F*, take the courſes or bearings, to every object neceſſary, with a compaſs or quadrant; as, to *a b c d e*, then take the courſe and meaſure the diſtance to another ſtation; as, *G*, from which take the courſes to the ſeveral objects as before; if the objects be not all viſible from each ſtation remove to a 3d, 4th, &c. Then by a circle from a fixed point repreſenting the firſt ſtation, *F*, draw a line a ſuitable length on each courſe; and lay off the diſtance its proper courſe to the ſecond ſtation, *G*, on this point place the circle in the ſame poſition as at the firſt ſtation, (viz. correſponding points parallel,) and draw lines their given courſes till they interſect thoſe from the firſt ſtation at the proper objects; as, *a b c d e*; the diſtances between the objects meaſured, by the ſame ſcale with which the ſtationary diſtance was meaſured, will ſhow the true diſtance between thoſe objects, and the courſes or bearings from one to the other by the circle will ſhow their true bearings.

Examples.

Required the ſituation of and diſtances between the following objects; viz. *a b c d e*; whoſe courſes or bearngs from the ſtation *F* are, North bearing Eaſt 82° to

S — E 85° 30′ to *b*. S — E 68° 30′ to *c*. S — E 76° to *d*, S — E 83° to *e* Then S — E 50° to ſta *G* 9,7 *ch* From thence, N — E 36 to *a*. N — E 69 20′ to *b*. N — E 79° to *c*. N — W 30′ to *d*. N — E 51° to *e* ?

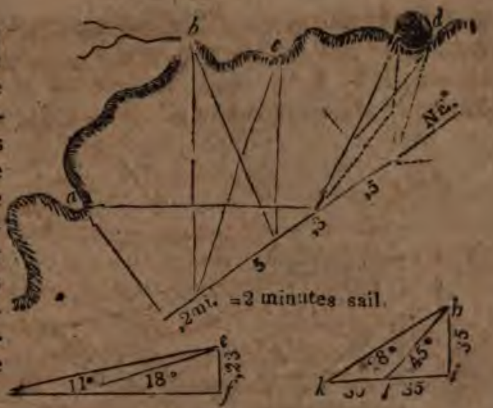


Scale of 10 chains per 1,625 in.

A. The diſtance from *a* to *b* 6,8 Ch. from *b* to *c* 6,8 &c. their ſituation as in the figure, the bearing or courſe from *a* to *b* from *d* to *c* is S — E 60°, that from *a* to *d* from *b* to *c* S — W 60° found by laying one ſide of a parallel on the two points and extending the other ſide to the centre *G* or *F*.

If a ſhip ſailed N E or N 45° E at the rate of 6 nots an hour = 1 mile in ten minutes and the following bearings, &c. were taken, to delineate the coaſt; viz N bearing W 50′ to a promontory, *a*, again in 2 minutes N — W 12° 30′ to *b*, N — E 9° to *c* (taken by two perſons) again in 5 minutes N — W 9° to *c*, N — W 33° to *b*, and in 3 minutes S — W 81° to *a*; at the ſame time the altitude of a mountain N — E 24° at *d* by a quadrant 11° and in 5 minutes the altitude of the mountain being 18° N — E 11°; what would be the height of the mountain and ſituation of thoſe objects?

The ſhip being 8 mi. nearer the ountain at the king of the 2d altude, determines e place where e line from the d angle (18°) inrfects the firſt 1°) which is e place to meaſre the perpendilar height of the ountain, *e f*.



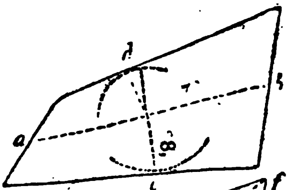
A. Height of the mountain is ,23 mi. *b* is ſituated N 24° 1,1 mi. from *a*, *c* S 89° E ,5 mi. from *b* &c as in the aught. The height of the mountain might have been dermined by one altitude knowing its diſtance from the veſſel. The altitude of a ſteeple, *hi*, at a certain diſtance from it, *k*, beg 28° and on advancing 30 yds. toward it, *l*, its altitude beg 45°; what is its height?* A. 35 yds.

o find the contents of a quadrilateral or four ſided figure, as a trapezium, rhomb, &c.

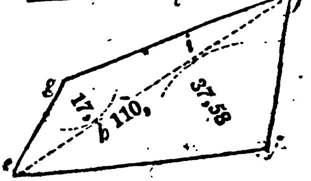
Rule. Draw a line from the middle of one end to the iddle of the other, which line is the mean length, and the erpendicular diſtance from each ſide to the middle of this ne is equal to the mean breadth of the figure; multiply the ean length and breadth of any figure together, and the oduct is the area or if the oppoſit ſides be of equal length, ultiply the perpendicular breadth by the length of one ſide, d the product is the area. (xy the area by the length for e ſolidity.)

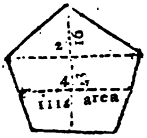
EXAMPLES.

length a. b. 79,
breadth d c 38
632
237
area 3002

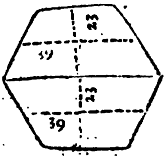


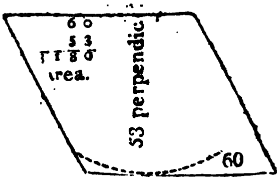
per. i. j. 37,58
per. g.h. 17
2)54,58
both per. 27,29=½
base of both 110,
3001,90 area
nearly as above.





39(+39=78)
23+23=46 23
234 234
156 156
area 1794 or 1794
it might be resolved into 4 triangles.



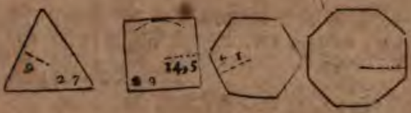


To find the contents of a regular polygon ; or, a figure havi equal ſides and angles.

RULE. Multiply the length of all the ſides by half t perpendicular from the middle of one ſide to the centre, half the ſides [or perimeter] by the whole diſtance from the middle of one ſide to the centre, and the product is the area.

27 x 3=81
2)9(= $4\frac{1}{2}$

324 } area.
$40\frac{1}{2}$ }



To find the area of a circle, the ſolidity of a cylinder, round timber, &c.

RULE.* Multiply the circumference of a circle by $\frac{1}{4}$ of its diameter, (or the diam. by $\frac{1}{4}$ the cir. or $\frac{1}{2}$ of each together,) or multiply the ſquare of the diameter by $\frac{11}{14}$ or ,7854, or the ſquare of the circumference by ,08 or ,0795775; in either caſe the product is the area: multiply the mean area of

---

[*] The first part of this rule only is perfect for finding the area, but any part is sufficiently exact for common use. There may be 50 or more methods of finding the area and different parts pertaining to a circle; but the chief that are useful are enumerated above

$\frac{22}{7}$ is ,001265 and ,08 is ,0004225 too large, supposing the diameter 1 and the cir. 3,14159 which is sufficiently exact for common use; though some have ascertained the proportion to 100 places of decimals & some farther, all agreeing as far as they go, that if the diameter be 1 the circumference will be 3,141592653589793238462643383879502884197169-39937510582097494459230781640528620899862803482534211706 79.

If the diameter of a circle be 4 (in. ft yd. &c.) the circumference= area=12,566368. If the circumference be 4 the diam. & area are each equal to 1,273241. If the diameter be 1 the circumference is 3,14159 =the area of that whose diameter is 2. If the diameter or circumference of one circle be double that of another the area is four times as large.

"If half the diameter of a circle be added to the whole diameter, this sum multiplied by the same half," the product will be the area of a hexagon inscribed within the circle.

What is the area of a hexagon whose circumscribing circle is 24 inches in diameter? 24+12=36, x 12=432 A.

any ſolid by its perpendicular length, and the product is the ſolidity.

The diameter of a circle is $\frac{7}{22}$ or $\frac{113}{355}$ of the circumference; ſo multiply the diameter by $\frac{22}{7}$ or 3,14159 for the cir. or the cir. by $\frac{7}{22}$ or ,31831 for the diameter.

EXAMPLES.

What is the ſolidity of a grindſtone whoſe diameter is 24 inches, cir. 75,4 and thickneſs 4 ?

75,4
6,=$\frac{1}{4}$ diameter.
452,4 area.
4, thickneſs.
in.
in. 1809,6 A.=1ft. 81,6

The mean diameter of a ſtick of timber being 2 ft and length 10 ft ; what is its contents ?

2 x 2=4 ſqr. of diameter x $\frac{11}{14}$=$\frac{44}{14}$=3$\frac{2}{14}$
10
ft 31$\frac{6}{14}$ A.

Or multiply $\frac{1}{4}$ of the circumference by itſelf and add $\frac{100}{366}$ of this ſum to itſelf ; or multiply $\frac{1}{3}$ of the circumference by itſelf and ſubtract $\frac{10}{35}$ of this ſum from itſelf ; for the area.

The circumference of a circle being 37,6 ; what is its area?

4)37,6(9,4 x 9,4=88,36 x $\frac{100}{366}$=24,14
24,14
112,50 area A.

What is the contents of a ciſtern whoſe mean circumference within the ſtaves is 8 ft. and length 5 ft. 9 in. ?

8 ft=96, x 96=9216
,08
737,28
5 ft 9 in = 69,
in. Wine gln.
1 W gl.=231)50872,32(220,226 A.

What is the diameter of a circle whoſe circumference is 342,75 yards?

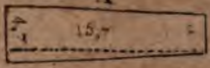
342,75
7

22)2399,25(109,056 yds. A.
3,14159

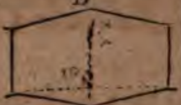
cir. 342,6 proof nearly.

To find the mean diameter or circumference of a caſk, ſtick, of timber, &c. whoſe ſides are ſtraight between the largeſt and ſmalleſt diameter; as A or B.

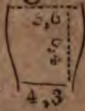
A



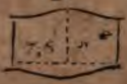
B



C



D



Multiply the difference between the largeſt and ſmalleſt diameter by $\frac{10}{19}$ or ,52752, this product added to the ſmalleſt diameter gives the mean diameter nearly. But when the ſtaves are curved as C or D multiply the difference by ,6 if more curved, by ,65 or ,7 (as ſeems requiſit) add this to the leaſt diameter; proceed in the ſame manner to find the mean circumference when required. Having the mean diameter or circumference proceed according to the preceding rule to find the ſolidity: or ſquare the mean diameter multiply the ſquare by the length within, and divide this product by 294 for wine gallons, by 359 for ale and beer gallons, or by $\frac{11}{14}$ for inches when the dimenſions are taken in inches.*

· · · · · ·

* "Multiply the largest and smallest diameters together, add $\frac{1}{3}$ of the square of their difference to the product, xy this sum by $\frac{11}{14}$ or ,7854 and the product will be the mean area; which xy by the length for the solidity."

What is the contents of a round ſtick of timber, as A whoſe largeſt diameter is 4,7 ft leaſt di. 3, ft and length 15,7 ?*

3,

dif 1,7 x $\frac{10}{19}$ = $\frac{17}{19}$ = ,9, + 3 = 3,9mean di x 3,9 = 15,21 ſquare, x $\frac{11}{14}$ = 11,95 mean area x 15,7 = 187,615 A.

What is the contents of a caſk of wine whoſe ſtaves are curved as D its head diameter within the ſtaves being 27 in. the middle diameter 33 and the length between the heads 36 in ? 114,65 W gls. A.

33 mid. diam.
27 head diam.

dif. 6, x ,6 xr = 3,6 + 27 = 30,6 mean di. x 30,6 = 936,36 ſquare x 36, length = 33708,96 ÷ 294 = 114,65

What is the mean diameter of a caſk as B 5,2 leaſt diameter and 7.4 largeſt ?

5,2

xr.
2,2 x ,53 = 1,166 + 5,2 = 6,366 A.

To find the area of a ſector or ſegment of a circle.

RULE. Multiply half the given arch or curve by the radius, (or diſtance from the arch to the centre of the circle,)

. . . . . .

* The solidity of tapering round timber may be thus found, nearly exact; xy $\frac{1}{3}$ of the middle circumference by itself, subtract $\frac{1}{4}$ of this product from itself for the mean area, or xy $\frac{1}{4}$ of the circumference by itself and add $\frac{1}{3}$ of this product to itself for the mean area; which xy by the length or the solidity.

What is the ſolidity of a round ſtick of timber as A whoſe girth or middle circumference is 12 ft. and length 15,7 ft. ?

3)12(4 x 4 = 16, x $\frac{1}{4}$ = 4, −16 = 12 x 15,7 = 188,4 ft. A.
or 4)12(3 = $\frac{1}{4}$ cir. x 3 = 9, ÷3 = 3, + 9 = 12, x15,7 = 188,4 ft A

What is the contents of a maſt 50 ft long whoſe middle circumference is 8 ft ?

4)8(2, x 2 = 4, x $\frac{1}{3}$ = 1$\frac{1}{3}$, + 4 = 5$\frac{1}{3}$, x 50 = 266$\frac{2}{3}$ A.
*or 3)8(*2$\frac{2}{3}$, + 2$\frac{2}{3}$ = 5$\frac{1}{3}$, + 1$\frac{7}{9}$ = 7$\frac{1}{9}$ = 7$\frac{4}{36}$, ÷4 = 1$\frac{28}{36}$ − 7$\frac{4}{36}$ = 5$\frac{12}{36}$ = 5$\frac{1}{3}$ x 50 = 266$\frac{2}{3}$ A.

If the timber be not tapering this method gives the contents about ,04 too large.

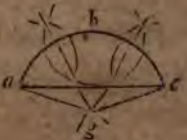
and the product is the area of that part of the circle; then if the ſegment given be leſs (as *a b c*) or greater (as *d e f*) than the area found as above, ſubtract from or add to the area accordingly.

The centre of the circle may be found, (when not given) as in deſcribing a circle to paſs through three points. Or by drawing two or more perpendiculars from different points in the arch till they met each other in the centre required.

To find the length of the arch-line (when not given) divide it into two equal parts and from the ſum of both chords ſubtract the chord of the whole arch and add $\frac{1}{3}$ of the difference to both the chords, (or to double the chord of half the arch,) for the length of the arch required. In the ſame manner the circumference of a circle may be found nearly.*

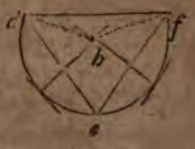
What is the area of a ſegment whoſe arch *a b c* is 6,4 and radius *a g* 2,5 the triangle *a g c* 1,68 to be deducted?

2)6,4 arch
——
3,2 = $\frac{1}{2}$ arch
2,5 = $\frac{1}{2}$ diameter
——
8,00 area of the ſector *a b c g*.
1,68 area of the triangle *a c g*.
——
6,32 area of the ſegment *a b c*.



In the ſegment *d e f* given the arch line 9,3 in. and the chord *d f* 4,8 in. to find the area. The centre found at *b* is 2,5 from the arch and ,7 from the chord.

4,65 = $\frac{1}{2}$ arch
2,5 = $\frac{1}{2}$ diameter
——
11,625 area of ſector, *d e f b*
1,68 —— triangle, *d f b*
——
A. 13,305 in.



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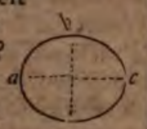
* Divide the ſquare of $\frac{1}{2}$ the chord by the diſtance from the middle of the arch to the chord, add this diſtance to the quotient, and their ſum will be the diameter of the circle.

To find the area of an ellipſis.

Rule. Multiply the longeſt diameter* by the ſhorteſt, multiply this product by ,7854 or $\frac{11}{14}$ and the laſt product will be the area Multiply the ſum of the two axes [the tranſverſe and conjugate] by 1,5708 for the circumference.

Examples

What is the area of an ellipſis whoſe longeſt diam. *a c* is 3,6
ſhorteſt dia. 2,5?
——
area A. 9,00



What is the contents of an oval box, whoſe longeſt diam. is 15 inches, ſhorteſt diam. 9 in. and depth 5 in ?
15 x 9 = 135, x 5 = 675 x $\frac{11}{14}$ = 530,357 + in. A.

To find the ſolidity of a pyramid, or ſolid body decreaſing regularly from the largeſt end to a point or vertex.

Rule. Multiply the area of the baſe by $\frac{1}{3}$ of the perpendicular height or length; or the whole length by $\frac{1}{3}$ of the area of the baſe (which is the mean area) and the product is the ſolidity.

Examples.

What is the ſolidity of a ſquare pyramid 48 ft high & each ſide of the baſe 13 ft ?
13
13
——
169 area of baſe
16 = $\frac{1}{3}$ of 48
——
2704 ſolidity. A. 13

What is the contents of a cone 48 high whoſe baſe diameter is 13 ?
13
——
169,
,7854
——
132,7326 area of baſe
16 = $\frac{1}{3}$ of height
——
2123,7216

To find the ſolidity of the fruſtum of a pyramid or a ſtick of timber, ſtone, &c. decreaſing regularly from the largeſt to the ſmalleſt end, but not to a point.

Rule. Multiply the ſide or (if circular the) diameter of the greater end by that of the leſs end, to this product add

* * * * * *

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*The longest diameter or length of an ellipse is called the transverse, and the shortest diameter or breadth the conjugate, they cross each other in the centre at right angles.

of the ſquare of the difference of the ſides or diameters, multiply this ſum, for the mean area, by ,433018 when the ends are trigons of 3 equal ſides, if a circle by ,7854

If a	tetragon	of 4	ſides	by	1,
—	pentagon	— 5	—	—	1,720477
—	hexagon	— 6	—	—	2,589076
—	heptagon	— 7	—	—	3,633959
—	octagon	— 8	—	—	4,828427
—	enneagon	— 9	—	—	6,181827
—	decagon	— 10	—	—	7,694209
—	endecagon	— 11	—	—	8,51425
—	dodecagon	— 12	—	—	9,330125

Multiply this product i.e. the mean area by the perpendicular length for the ſolidity.

Note. If the side of one of the above polygons be squared and the square multiplied by its xr, the product is the area.

Examples.

Required the ſolidity of the fruſtum of a triangular pyramid, one ſide of the larger end being 15 and of the leſs 6, its length 40 ?

15 15
6 ſides 6
— —
9 dif. of ends 90
9 27
— —
3)81 ſqr. 117
— ,433 xr
27 = $\frac{1}{3}$ of ſq. —
mean area 50,661 x 40
=2026,44 A.

What is the contents of a ſquare ſtick of timber 14 ft long 2 ft ſquare at the largeſt end and 1,5 at the leſs end ?

1,5
2 2,
1,5 —
— 3,0
,5 x ,5 = ,25, x $\frac{1}{3}$ = ,08 +
—
mean area 3,08
14,
—
ſolidity 43,12 ft A.
1, being the xr of the ſquare or tetragon.

As the difference between the two ſides is to the perpendicular height ſo is the longeſt ſide (i e. the ſide of the largeſt *end of the* fruſtum) to the length at which it would end in a *point.*

What is the ſolidity of a ſtone whoſe perpendicular height is 10 ft the ends hexagons each ſide of the larger end 14 in. and of the leſs 6 in. ?

14 14
6 6
— —
8x8=64x$\frac{1}{3}$=21,33 (=$\frac{1}{3}$ ſqr. of dif.)+ 84=105,33
2,589 x*r* of hex.

in. 272,69937 mean area.
10
ft.

No. of inches in one ſqr ft. is 144)2726,9937(18,937+A.

"Or ſquare $\frac{1}{4}$ of the middle girth of a ſquare tapering ſtick of timber (whoſe angles are all equal ;) then as 144 (inches) is to that product in inches ; ſo is the length in feet, to the contents in feet."

What is the contents of a tapering ſquare ſtick of timber, whoſe length is 30 ft and middle girth 40 inches ?
As 144 : ($\frac{1}{4}$ girth=10, x 10=) 100 :: 30 ft : 20,83 ft A.

To find the ſuperficial and ſolid contents of a globe or ſphere.

RULE. Multiply the circumference by the diameter, and the product will be the ſuperficial contents ; which multiplied by $\frac{1}{6}$ of the diameter will produce the ſolidity.

Or, multiply the cube of the diameter by $\frac{11}{21}$ or ,5236 and the product will be the ſolidity.

Ex. What is the ſuperficial and ſolid contents of a globe, whoſe circumference is 22 in. and diameter 7 ?

22
7
surface 154 in A.
7 x $\frac{1}{6}$= 1,16
solidity 178,64 in. A
or 7 x 7 x 7=343
11
21)3773(179,66 sol. A.

In the frustum a b c d
3,2
3,2
10,24 to 3 times
3 the sqr
of $\frac{1}{2}$ the base a c
30,72
4,84 add sqr of height (b d
35,56
2,2 xx by height
78,232 [illegible]
]

To find the ſolidity of the fruſtum or ſegment of a globe, or a part cut off parallel to a plane.

Rule. To three times the ſquare of $\frac{1}{2}$ the baſe add the ſquare of the thickneſs or height; multiply this ſum by the height, this product by ,5236, and the laſt product is the ſolidity.

To find the ſuperficial and ſolid contents of a ſpheroid.

Rule. Multiply the leaſt middle circumference by the longeſt diameter and the product is the ſuperficial contents; which multiplied by $\frac{1}{6}$ of the ſhorteſt diameter will produce the ſolidity.

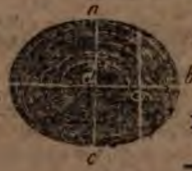
Or, multiply the ſquare of the ſhorteſt diameter by the longeſt diameter or length, this product by $\frac{11}{21}$ or ,5236, and the laſt product will be the ſolidity.

Examples.

What is the contents of a ſpheroid whoſe middle circumference is 22, diameter 7 and length 9,6?

7
7
——
49
9.6
——
470.4
,5236
————
246,30144 sol. A.

9.6
22.
——
211,2 sur. A
7x$\frac{1}{6}$= $\frac{7}{6}$=$\frac{1}{6}$ of di.
————
6)1478,4
——
246,4 sol. A.



In the frustum or half a b c.
6
6 to 4 times the
——
36 sqr of mid. di.
4
—
144
49+sq of base a b
——
193 xy by ,7854
,7854 and
————
151,5822 by $\frac{1}{3}$ of
,8 length d b
——————
121,26576=solidity of a b c d nearly.

To find the ſolidity of the middle zone of a globe or ſpheroid, or the remaining part after two equal ſegments are cut off, parallel to the circumference.

Rule Multiply the ſquare of the middle diameter by 2, to the product add the ſquare of the end diameter, multiply *this ſum by ,7854,* this product by $\frac{1}{3}$ of the length, and the *laſt product will* be the ſolidity.

If the middle diameter of a zone be 20 in, the end diameters each 16 in, and the length 12 in ; what is its ſolidity ?

$\overline{20 \times 20 \times 2 + 16^2} \times ,7854 \times 4 = 3317,5296$in ſolidity. A.

To find the ſolidity of a parabolic conoid.

Rule. Multiply the area of the baſe by one half of the perpendicular height, and the product is the ſolidity required.

If the diameter of the baſe of a parabolic conoid be 20 and its height 20 ; what is its ſolidity ?

$20 \times 20 \times ,7854 \times 10 = 3141,6$ ſolidity. A.

To find the ſolidity of a parabolic ſpindle.

Rule. Multiply the area of the middle by $\frac{8}{15}$ of its perpendicular length, and the product will be its ſolidity.

If the middle diameter of a parabolic ſpindle be 20 and its length 60 ; what is its ſolidity ?

$60 \times \frac{8}{15} = \frac{480}{15} = 32.$ $20 \times 20 \times ,7854 \times 32 = 10053,12$ A.

To find the ſuperficial and ſolid contents of the five regular bodies ; viz.

The tetrahedron, hexahedron or cube, octahedron, dodecahedron and eicosihedron, or bodies contained under so many equal and equilateral sides or bases as their names severally represent ; viz. the first under 4, the second 6, the third 8, the fourth 12, the fifth 20, the sides of the 1st, 3d and 5th are trigons of the second squares, and the sides of the fouth are pentagons ; the sides may be considered as the bases of so many pyramids whose common vertex is the centre of the body.

Rule. Multiply the area of one of the ſides or baſes by the number of ſides, this product is the ſurface or area of the figure ; which multiplied by $\frac{1}{6}$ of the diameter, or diſtance from the middle of one ſide or baſe to that of the oppoſit ſide, will produce the ſolidity ; (the tetrahedron or triangular pyramid excepted.)

Or multiply the cube of one ſide of a tetrahedron by	,11785 for the ſolidity.	and multiply the ſquare of one ſide by 1,73205 for the ſurface.
of a hexahedron, by	1,	6,
— *octahedron* —	,4718	3,464
— *dodecahedron* —	7,663119	20,6457
— *eicoſihedron* —	2,181695	8,66025

Note. The above xrs are the solid and superficial contents of their respective bodies when the sides or lines are 1.

EXAMPLES.

The ſide or line of an octahedron being 3 its per. 2,5+ and the diameter 2,5 ; what is its contents ?

This body contains 8 triangular pyramids the baſe of each is 3 on a ſide and each pyramid is (2,5 ÷ 2 =) 1,25 high therefore.

2)2,5(1,25 = $\frac{1}{2}$ per. of each ſide	or 3x3x3 = 27x,4718 =
3 ſide of each ſide	12,7386, ſolidity. A.
3,75 area of each ſide	3 x 3 = 9 x 3,464 =
8 No of ſides	31,176 ſurface. A.
30,00 whole ſurface A.	
6)2,5(=,41666 $\frac{1}{6}$ of diam.	
12,4998+ ſolidity A.	

The ſide of a dodecahedron being 3 and its diameter 6,68 ; what is its ſurface and ſolidity ?

In this ſolid there are 12 pyramids whoſe baſes are pentagons, therefore,

3 x 3 = 9, 1,72 x*r* of pentagon	3 x 3 x 3 x 7,663 =
9	206,9 + ſol.
15,48 area of each ſide	3 x 3 x 20,6457, =
12 No. of ſides	185,8113 area.
185,76 whole ſurface A.	
6)6,68 = 1,1133 $\frac{1}{6}$ of height or di.	
206,805 + ſolidity A.	

To find the ſolidity of a body or part whoſe dimenſions cannot be eaſily taken.

RULE. Put the body or part, (wet or dry as ſeems requiſit,) into any regular veſſel with water, (ſand, grain, &c.) ſufficient to cover it, and while immerſed find how much it raiſes the water, or how much the water falls when it is taken out ; *which multiplied by the area of the veſſel will produce the ſolidity.*

Or, from the real weight of the body ſubtract, its weight when it or the part required is ſuſpended in the water from the balance by which it is weighed; multiply this difference by ſuch a number as will reduce water from weight to ſize; (which number or numbers may be found in table 3d or 4th. as 1 lb avor. of water=27,66+ in. 1 oz apoth.=1,897 in.)

By immerſing a body the area or ſolidity of a regular tight veſſel may be eaſily found; knowing the ſolidity of the body immerſed, which will be equal to the quantity of water raiſed.

Alſo the quality of gold, ſilver, &c. may be tried as above, for if the weight of the metal and its loſs by being balanced in the water be accurately found, it will ſhow how much heavier the metal is than its bulk of water. Very fine gold is 19+ times and ſilver 11 times heavier than its bulk of water, gold 18+ and ſilver $10\frac{1}{2}$ times is ſtandard.

EXAMPLES.

A body being covered with fluid in a veſſel 18 inches ſquare and on taking out the body the fall of the fluid was 9 inches; what is its contents?

18 x 18=324 veſſel's area, x 9=2916 in. A.

Or 1728)2936(1,6875 ſolidity in ft. A

A ſmall body being held with a part of it in a cup of water 3 inches in diameter, and while it was in, the water was raiſed 1 inch; required the ſolidity of the part?

3 x 3 x ,7854=7,0686 in. A.

If a guinea were ſuſpended by a hair from the underſide of a ſcale, and 129 grains juſt counterbalanced it in air and while immerſed in pure water requires 7,25 gr. to be in the ſcale over it to reſtore the equilibrium; what is its ſolidity and how many times heavier than its bulk of water?

its loſs in water $7\frac{1}{4}$ x ,00395 in. 1 gr. water
7,25,

in. ,0286375 ſol. A=49,48'+

aerial wt. 129 gr. ÷7,25=17,793 times heavier than water A.=good gold.

A dozen of apples being put into a wine gallon meaſure, *which* was afterwards filled with 3 pints of ſand while the apples were in it; required their contents?

8 pints=1 gln.—3=5 pints A.=129 in.

What is the area and contents of a ciſtern 5 ft high, if a body, 1 ft ſquare, being ſunk 6 inches into the water within the ciſtern, raiſe it 1 inch, the ciſtern being equal in all places?

1 inch in height contains ,5 ft or 864 in.
12
—
ſo 1 ft=12 in. high=6,0 ft area. A.
5
—
ft 30, ſolidity. A.

If the real weight of a dollar be 409 gr. and when ſuſpended in water, it looſe 38,95 gr. of its weight; required whether it be good or counterfeit?

loſs 38,95)409(10,5 good or ſtandard ſilver A.

In a ſimilar manner the ſpecific gravity of all bodies may be found. The ſpecific gravity of bodies is their weight compared with their bulk or ſize; as, if a body, of the ſame ſize of another, weigh twice as much it is ſaid to have twice the ſpecific gravity; ſo very fine gold has 19+ times the ſpecific gravity of water.

If a piece of ſteel weighing 7,85 lb be immerſed in water, and looſe 1 lb of its weight while ſuſpended therein; what is its ſpecific gravity?

loſs 1)7,85(7,85 times heavier than water A.

If a dollar weighing 410 gr be immerſed in a ſpirituous liquor and looſe 36 gr. of its weight while ſuſpended therein; whether is the ſpirits proof or not?

36)410(11,3888+ =proof ſpirits A. which are (by T. 16) ſo many times lighter than ſilver.

As the areas or ſurfaces of all ſimilar figures are in proportion to each other, as the ſquares of their like ſides or dimenſions; alſo as the ſolidities of ſimilar bodies are to each other, as the cubes of their ſimilar dimenſions; therefore

To form a rule for meaſuring or finding the contents of all ſimilar figures of any ſize or figures whoſe angles or corners are the ſame and equal in number, all the ſimilar dimenſions proportional.

Rule. Find the true contents of one of thoſe ſimilar fi ures by ſome known method, by immerſing the body in ter, by weight, or by the parts ſeparately, or by coverings, &c. (for the area;) then ſquare and cube a ſuitable dimenſion, and by the ſquare divide the true area or ſurface and by the cube the ſolidity, (of that figure whoſe dimenſion was ſquared or cubed for the diviſor,) and give the ſquare of this dimenſion and its quotient as the numbers to be multiplied together to produce the area or ſurface; and the cube and its quotient as thoſe to produce the ſolidity, (of any figure of the ſame ſhape of that by which the experiment was made.)

Or, divide the ſquare of any ſuitable dimenſion by the known area, and the cube by the ſolidity, and give the ſquare of the ſame dimenſion as the dividend and the quotient as a diviſor for the area, and the cube as a ÷*nd* and its quotient as a ÷*r* to find the ſolidity.

NOTE. The product of two dimensions may be used as the square and of three as the cube in all respects.

EXAMPLES.

By what rule may all figures of the following ſhape be meaſured; viz. having 8 ſides all equal and equilateral triangles; one of them, whoſe ſide is 6 inches being meaſured, and contained (15,6 on each ſide x 8 =) 124,71 ſuperficial inches, and (being immerſed in water found to contain,) 101,83 cubic inches?

one ſide 6 x 6=36)124,71(3,464+
6 x 6 x 6=216)101,83(,4714+

the qts are the contents if the figure's ſide be 1.

or contents { 124,71)36,0000(,2886+
101,83)216,0000(2,12118+

here the firſt quotient is the ſquare of each ſide if the area be 1, and the ſecond qt. the cube if the ſolidity be 1. Therefore the rule may be

Multiply the ſquare of one ſide by 3,464 and the product will be the area or ſurface; or divide by ,2886 and the quotient will be the area, and multiply the cube of one ſide by ,4714 or divide it by 2,12118 for the ſolidity. A.

Ex. What is the ſolidity of a body ſimilar to the above, *the length* of whoſe ſide is 2?

ſide 2 x 2 x 2=8 x ,4714=3,7712 A.
or 2 x 2 x 2=8 ÷2,12118=3,7714 A.

The farther the quotients are carried the nearer exact the anſwers will be.

As 114,6 wine gallons will fill a caſk of a common curve, whoſe middle diameter is 33 in. and diſtance from the middle of one ſide, to the lower part of each head within is 35 in. and length within 36 in.; in what manner may the contents of any other caſk of the ſame ſhape be found?

33 x 35 x 36=41580)114,60000(,002515

or 33 x 33 x 35=38115)114,600(,003+

114,345

Rule. Multiply the middle diameter by the diſtance from the middle of one ſide to the lower part of each head, multiply this product by the length within (all taken in inches) multiply the laſt product by ,0025|156, and this product will be wine gallons.

Or multiply the ſquare of the middle diameter by the diſtance from the middle of one ſide, to the lower part of each head xy this product by ,003 for wine glns if the dimenſions were taken in inches.

Ex. What is the contents of a caſk, whoſe middle diameter is 22 inches and diſtance from the middle of one ſide to the lower part of each head 23,33 inches, of the ſame ſhape of the above mentioned?

23,33 x 22 x 22=11291,72 x ,003=33,87 wine gln. A.

If the circumference of a tree be 1 ft. and its contents 4 ft; how may the contents of any other ſimilar tree be found?

circumference 1 x 1 x 1=1)4(4.

Rule. Multiply the cube of the circumference taken in its proper place by 4 and the product will be the ſolidity; (provided its height and branches be in proportion to thoſe of the tree whoſe circumference is 1 and ſolidity 4, as its circumference is to that.) A.

Ex. On the above conditions what is the contents of a tree whoſe circumference is 2 ft?

2 x 2 x 2 x 4=32 ft. A.

In a ſimilar manner may rules be formed to determine the ſize, weight, value or ſurface of any ſubſtance whatever.

For more particulars in reducing figures, dimenſions, &c. to thoſe of different kinds; ſee numeration of ſpace, where the proportions between figures, dimenſions, &c. of different kinds, (i. e. the values of one in another,) are given in decimals or vulgar fractions; Therefore,

To reduce one figure, dimenſion, &c. to another.

RULE. Multiply the given part by the proportion which it bears to that which is required, and the product is the anſwer: the parts given may uſually be found at the left of the tables and thoſe required at the right or top; then either at the right or left of the given and under or at the left of the required part, is the proportion, value or multiplier required; (invert any multiplier to reduce the part written over or at the right of it, to that at the left;) if the multiplier be a vulgar fraction, multiply by the upper and divide the product by the lower part of the fraction, whether it be a ſimple fraction or compounded of two or more parts; thus multiply the given dimenſion by the firſt part of the dividend of the fraction and the product by the ſecond part, &c. and divide the laſt product by the firſt part of the diviſor divide this quotient by the next part, &c. and the laſt quotient is the anſwer, or part required.

EXAMPLES.

The circumference of a circle being 20 in. to find the area.

pro. $\frac{\text{cir.}\ *}{12,566+}$ i. e. xy the cir. by itſelf and ÷e by 12,566;

$\frac{20\times20}{12,566}=31,83$ area. A.

The circumference of a circle being 10 inches to find the ſide of an equal ſquare, (i e a ſquare containing as much as the circle) and the ſide of a ſquare inſcribed.
(The xr oppoſit circumference and under ſide
of an equal ſquare is ,282.)

,282 x 10=2,82 A. side of equal ſquare.

,225 x 10=2,25 A. of ſquare inſcribed.

* * * * * *

[*] Multipliers of this kind are seldom perfect, but generally more exact than dimensions can be taken.

The circumference of a cylinder being 10 ft its length 15 ft ; what is its ſolidity, and the diameter of a globe of the ſame contents ?

$\frac{10 \times 10 \times 15}{12,566} = \frac{1500}{12,566)}1500,(119,37$ ſolidity. A.

$119,37 \times \frac{1}{,5236 \text{ \& 3d root sq'd}} = \sqrt[3]{119,37 \div ,5236} = 6,11$ globes diameter. A.

The diameter of a circle being 4 ; what is its area ?

pro. $\frac{\text{di.}}{1,273} = \frac{4 \times 4}{1,273)}16,(12,566$ A.

The area of the end of a log being 100 inches ; how large will it be after it is hewn ſquare ; i. e. what is the ſide of the ſquare inſcribed ?

pro $\frac{,6364}{\text{2d root}}$ i. e. xy by ,6364 and divide the product by its ſquare root.

$\dot{}\ \dot{}$

,6364 x 100=63,64

ſuppoſe 8)63,64(7,955

8,

(7,977)63.64(7,977) 2)15,955(7,977+ A.

each ſide of the ſquare will be 7,977+ and its area 63,62+

What is the ſolidity of a triangular priſm, each ſide of the ends being 48 in. and its length 108 ?

$$\begin{array}{r} 248832 \\ 5 \\ \hline 11)1244160 \\ \hline 113105\frac{5}{11} \end{array}$$

is the ſolidity of a cube or oblong ſolid of which the priſm is $\frac{5}{11}$

$$\begin{array}{r} 48 \\ 48 \\ \hline 2304 \\ 108 \\ \hline 248832 \\ \frac{5}{11} \\ \hline 113105\frac{5}{11} \text{ A.} \end{array}$$

What is the contents of a triangular pyramid 15 ft high, whoſe baſe is 5 ft on each ſide ?

5 x 5 x 15=375=ſolidity of cube of which the pyramid is $\frac{13}{90}$

39,72 ft A.

What is the ſolidity of a cone or circular pyramid, 15 ft high, and 5 ft in diameter at the baſe ?

5 x 5 x 15 x $\frac{5}{19}$=98,6 A.

If the diameter of a ſpherical balloon be 50 ft ; how many feet of air does it contain ?

50 x 50 x 50 x $\frac{11}{21}$=65476,19 ft A.

What is the ſolidity of a parabolic conoid, whoſe baſe is 2,9 ft in diameter, and height 6 ft ?

2,9 x 2,9 x 6 x $\frac{11}{28}$=19,823 ft A.

or 2,9 x 2,9 x $\frac{11}{14}$=39,647 ſolidity of cylinder.

$\frac{1}{2}$

19,823 ft A.

If the weight (in avordupois ounces) of any thing be divided by its ſpecific gravity the quotient will be its ſolidity or magnitude ; which multiplied by its ſpecific gravity produces its weight.

What is the ſolidity or ſize of 100 lbs of iron ? (7,645 oz =,001 f.)

100
16

1600 oz ÷7,645=209,287 thouſandths of a ft
,001
or ,209287 ft A.
1728

or 361,649836 in.

What weight of lead, $\frac{1}{120}$ ft thick will cover the roof of a houſe, whoſe area is 3000 ft ?

3000 x $\frac{1}{120}$=25 cubic ft.
1000

(,001 ft=11,325 oz.) 25000 x 11,325=283125 oz A.

If the weight of any thing be divided by its ſize the quotient will be its ſpecific gravity.

If ,003 of a cubic ft of ſilver weigh 31,606 oz. ; what is *its ſpecific* gravity ?

*,003)31,606(*10535,33 oz. weight of 1 ft or 10,53533 oz weight of ,001 ft. A.

To find the tunnage of veſſels.

Rule. "Multiply the length of the keel by the breadth on the main beam, multiply this product by the depth, (all taken in feet,) and divide the laſt product by 95 for the tunnage of a ſingle-decked veſſel if the veſſel be double-decked, half the breadth on the main beam is uſed as the depth (above) in finding the tunnage."

Note The weight of a ships burthen is computed to be half the weight of water she can hold, of the same kind she swims in. A vessel will carry about $\frac{3}{100}$ more in the ocean than in pure water.

Examples.

What is the tunnage of a ſingle decked veſſel whoſe length is 60 ft breadth 20 ft and depth 8 ft ?

$$60 \times 20 = 1200$$
$$8$$
$$95)9600(101\tfrac{5}{95} \text{ tuns. A.}$$

What is the tunnage of a double decked veſſel, whoſe length is 65 ft and breadth 21,4 ft ?

$$65 \times 21{,}4 = 1391{,}$$
$$2)21{,}4(\quad 10{,}7$$
$$95)14883{,}7(156{,}67 \text{ tuns. A.}$$

"*To find the government tunnage.*

Rule. "If the veſſel be double-decked, take the length thereof from the fore part of the main ſtem, to the after part of the ſtern poſt, above the upper deck; the breadth thereof at the broadeſt part above the main wales, half of which breadth ſhall be accounted the depth of ſuch veſſel, and then deduct from the length three-fifths of the breadth, multiply the remainder by the breadth and the product by the depth, and divide this laſt product by 95, the quotient whereof ſhall be deemed the true contents or tunnage of ſuch ſhip or veſſel; and if ſuch ſhip or veſſel be ſingle-decked, take the length and breadth as above directed, deduct from the ſaid length three-fifths of the breadth, and take the depth from the underſide of the deck plank, to the ceiling in the hold, then multiply and divide as aforeſaid, and the quotient ſhall b deemed the tunnage."

What is the government tunnage of a double-decked veſſel, whoſe length is 75 ft. 6 in. breadth 23 ft 4 ?

length 75 6 −14 ft ($\frac{3}{5}$ of breadth)=61 6 x 23 4 breadth= 1435, x 11 8 ($\frac{1}{2}$ br.)=16741 8, ÷95=176$\frac{21}{95}$ tuns A.

"*To take the dimenſions of veſſels, by carpenter's meaſure.*

Meaſure the length by beginning at the aftermoſt ſide of the ſtern poſt, in a line with the rabbet of the garboard ſtreak, and meaſure by the rabbet continuing this line forward until a ſquare from it will meet the fore part of the main ſtem, immediately under the bowſprit; this gives the extreme length. The width is taken by meaſuring from outſide to outſide at the main beam, or greateſt breadth at the bottom of the black ſtreak juſt above the main wales. The depth of hold is taken from the under ſide of the deck at her main hatch-way to the top of the limber plank by the kelſon. The above dimensions are taken in feet.

To find the tunnage from the above admeaſurement.

Deduct $\frac{3}{5}$ of the breadth from the extreme length, then multiply the remainder by the whole breadth, and if a ſingle deck multiply this by her depth of hold; if a double deck, take half the breadth of beam for the depth of the hold and multiply as for a ſingle deck."

What is the tunnage of a veſſel whoſe extreme length is 90 ft, breadth 25 and depth of hold 10 ?

As a Single Decked Veſſel.

extreme length 90
−t $\frac{3}{5}$ of beam 15
—
75
xy by breadth 25
——
1875
xy by depth 10
——tuns.
÷*e by 95*)18750(197$\frac{35}{95}$

As Double Deck, of the ſame dimenſions.

90
15
—
75
xy by breadth 25
——
1875
xy by $\frac{1}{2}$ breadth 12,5
—— tuns.
÷e by 95)23437,5(246

EXCHANGE.

Exchange is the bartering or ſelling of the money or value of one country for the like value in the coin, weight or meaſure of another.

The value of any ſubſtance is uſually computed in money, familiar to the owner, of which, as of weights and meaſures, there are two kinds, *real and imaginary.*

Real money, weights and meaſures are ſuch as are known by ſight or their real intrinſic value, gravity or capacity; as, a dollar, pound, gallon, &c. and their real diviſions.

Imaginary money, weights and meaſures are ſuch as are known only in imagination, of which there are no real ſtandard, and are uſed only in computation; as, a £, Cwt, Chaldron, &c. in America.

Courſe of Exchange is the current price of the money of one country in another, and is ſometimes more, and ſometimes leſs than its juſt value; of this *courſe* there are uſually tables publiſhed daily in commercial cities, (commonly favorable to their citizens.)

Par is the juſt rate or courſe of exchange, found by comparing the money of one country with that of another, eſtimating each at its intrinſic value; at which rate the following tables of exchange are principally calculated; but the changeable ſituation of the money of ſeveral countries render it impoſſible to aſcertain the true par.

To exchange the Coins, Weights and Meaſures of one country to thoſe of any other.

Rule. Multiply the *given ſum* by the proportion which *that* Coin, Weight or Meaſure bears to the one which is required, and the product is the anſwer.

If the number to be reduced be a compound, uſe the proportion between the higheſt denomination given and the de- *omination* to which it is to be reduced, and proceed as with ompound numbers and the ſmall denominations of the an ſwer will be of the ſame diviſion of the given number; or reduce all to the ſame denomination; or find the value of each denomination ſeverally, & their ſum will be the anſwer.

TABLES OF EXCHANGE,

In which the *proportions* between the United States' currency and thoſe of moſt of the kingdoms and provinces of the world, are given.

GREAT-BRITAIN.

Sterling Money, reduced to	$.	*Cts.*
4 farthings= 1 penny, =	$\frac{1}{54}$	1,85
12 pence 1 ſhilling,	$\frac{2}{9}$	22,22
20 ſhillings, 1 pound,	$\frac{40}{9}$	444,44
21 ſhillings, 1 guinea,	$\frac{42}{9}$	466,66
5 ſhillings 1 crown,	$\frac{10}{9}$	111,11

Examples.

Change 432 d to $.
54)432(8, A.

Change 200 s to $.
200 x $\frac{2}{9}$=44,44+ A

Change 42 £ 16 s 6 d to $.
40 40
9)1713 0 0
190 $\frac{6}{20}$ $\frac{8}{12}$
190,33+ $ A

Change 454,4 $ to ſterling.
9
40)4089,6(102,24 £ A.
or 20
s 4,86
1
d 9,6

IRELAND.

	$.	*Cents.*
4 farthings=1 penny,=	$\frac{2}{117}$=	1,70
12 pence 1 ſhilling,	$\frac{8}{39}$	20,51
20 ſhillings 1 pound,	$\frac{160}{39}$	410,25

Reduce 120 d. x
$\frac{2}{117}$)240(2,051 $ A.
Reduce 200 $. x $\frac{39}{8}$
=975 £. A.

GERMANY.

HAMBURG, BREMEN, LUBEC, ALTONA, &c.

	$.	*Cts.*
12 deniers or 2 grotes=1 ſhilling lub or ſtiver of Hamburg =	$\frac{2}{105}$=	2
16 ſhillings lubs or ſtivers=1 mark banco, of Hamburg,	$\frac{1}{3}$	33$\frac{1}{3}$
12 grotes or pence=1 ſhilling flemiſh,	[illegible]	12
20 ſhillings=1 pound flemiſh,	[illegible]	240

3 marks=1 rix dollar of Hamburg,	$\frac{1}{1}$	100
$2\frac{1}{4}$ marks or 72 grotes=1 rix dollar of Bremen,	$\frac{3}{4}$	75

2 trilings=1 sexling.=,86 cts 2 sexlings 1 fenning, 1,74 c. 2 marks 1 selt dollar, 66,666 c. 4 marks 1 silver ducatoon, 133,333 c. $6\frac{1}{4}$ marks 1 ducat 208,333 c.

NOTE. The bank money is more valuable than the current money of Hamburg by 20 or 25 per cent, or the agio.

280 pounds=1 ſhippound of Hamburg=$300\frac{3}{5}$ lb avordupois. 1 pound in Bremen=$\frac{11}{10}$ lb avor. 1 ell=$\frac{5}{8}$ yard in U.S. 1 ring of ſtaves=240. the Bremen laſt=80 buſhels in U. S.

HOLLAND AND ZEALAND.

AMSTERDAM, ROTTERDAM, MIDDLEBURG, FLUSHING, &c.

	$.	Cts.	
8 pennings=1 grote,	$\frac{1}{100}$	1	Reduce 110 gr.
2 grotes or 16 pen.=1 ſtiver,	$\frac{2}{100}$	2	to $. A. 1,1 S
20 ſtivers or 40 gr.=1 gil.or flor	$\frac{4}{10}$	40	
12 grotes=1 ſhilling,	$\frac{12}{100}$	12	Reduce 247 ſti-
20 ſhilns or 6 guilder 1 pound fl,	$\frac{24}{10}$	240	vers to $ x 2=
$2\frac{1}{2}$ florins or 50 st.=1 rix dollar,	$\frac{1}{1}$	100	A 4,94 $.

16 pennigens or 2 grotes=1 stiver,=2 cts. 6 stivers 1 scalin, 12 c. 1 guilder or florin 40 c. 1 rix dollar 100 c. 1 dry guilder 120 c. 1 ducat 210 c. 1 silver ducatoon 120 c. 1 gold ducat 800 c. 1 sovereign 600c.

1 lb in Amſterdam=$\frac{219}{200}$ lb Avor. 1 ell=$\frac{3}{4}$ yard.

DENMARK.

COPENHAGEN, BERGEN, &c.

	S.	Cts	
1 ſkilling=	$\frac{1}{96}$	1,04	Reduce 446 sk.
96 ſkillings=1 rix dollar=	$\frac{1}{1}$	100,	to $ x $\frac{1}{96}$=4,64

16 skillings or 2 druggens=1 marc=16,666 cts 20 skellings 1 rix marc, 20,833 c. 1 rix ort 25 c. 1 crown 66,666 c. 1 ducat 183,333 c. 1 hat ducat 233,333 c.

1 pound =	$\frac{100}{96}$ *lbs avor*	Reduce 40 lbs to avor, x $\frac{100}{96}$
16 lbs=1 liſpound =	$\frac{100}{6}$	41,66
20 lsp=1 ſhippound	$\frac{1000}{3}$	13 ſhipp 8 lſp. 4 lbs. at 100 rix d. pr ſhippound
		134 $\frac{1}{20}$ 0 A., or 134,[illegible]

In Holstein and Sleswick 1 rix dollar=3 marks current of Hamburg. *Specie dollar of* Holstein and Sleswick 3 marks 12 d. current.

FLANDERS, NETHERLANDS.

BRUSSELS, ANTWERP, GHENT, OSTEND, &c.

	\$.	*Cts.*
12 grotes=1 ſhilling=	$\frac{12}{100}$	12
$3\frac{1}{3}$ ſhil or 40 gr 1 gilder=	$\frac{4}{10}$	40

1 lb $\frac{100}{96}$ lbs avor. 1 pot=$\frac{73}{200}$ gallon in U. S.

1 ell=$\frac{74}{100}$ yards in U. S.

RUSSIA.

PETERSBURG, MOSCOW, ARCHANGEL, &c.

	\$.	*Cts.*
1 copec=	$\frac{55}{10000}$	$,5\frac{1}{2}$
100 copecs=1 ruble=	$\frac{55}{100}$	55,

Reduce 450 copec to \$. 2,475 A.

2 poluscas=1 danusca,=,275 cts 3 copecs 1 altin, 1,65 c. 10 copecs 1 grievener, 5,5 c. 1 polpotin=13,75 c. 1 poltin, 27,5 c. 1 servonitz, 110 c.

	lbs avor.
96 zollotnicks=1 lb,=	$\frac{78}{80}$
40 pounds=1 pood,=	$7\frac{1}{2}$
10 poods=1 barquet=	$7\frac{10}{2}$
1 arſheen=$\frac{7}{9}$ yard in U.S.	

Reduce the weight of the great bell of Moſcow being 1250 barquets 24 pounds to avor. wt.

```
     1250   0  24
               710
   ---------------
  2)887542  6   0
   ---------------
  lb.443771 3/10  A or 198 tuns
                        251,3 lbs.
```

FRANCE.

PARIS, LYONS, MARSEILLES, BOURDEAUX, DUNKIRK, LISLE, BAYONNE, &c.

	\$.	*Cts.*	*Francs.*
12 deniers=1 ſol,=	$\frac{37}{4000}$	$,9\frac{1}{4}$	$\frac{80}{1620}$
20 ſols=1 livre tornois,=	$\frac{37}{200}$	$18\frac{1}{2}$	$\frac{80}{81}$
3 livres=1 crown of exch.	$\frac{111}{200}$	$55\frac{1}{2}$	$\frac{240}{81}$
			livres.
1 centime=	$\frac{1}{534}$	,187	$\frac{81}{8000}$
100 centimes=1 franc=	$\frac{100}{534}$	18,34	$\frac{81}{80}$

Reduce 100 livres to \$.

18,5 A.

Reduce 8 livres, to francs.

81 x 80/81)=80 fr. A.

15 deniers=1 petard,=1,15 cts. 20 petards or 1½ peittes 1 florin, 22,14 c. 6 livres 1 ecu or 1 crown, 110 cts. 10 livres 1 pistole, 185 c. 1 ducat 194¼ c. 1 louis d or 444,4 c. 1 guinea 466,2 c. 1 moeda 638 c.

1 gram=15,444 grains troy. 10000 grams 1 myriogram =22 lb. 1 oz. 15 dr. avor. 1 decaliter=2 gallons 148,28 cubic inches. 1 meter=39,371 inches in U. S.

SPAIN.

MADRID, CADIZ, SEVILLE, MALAGA, BILBOA, BARCELONA, SARAGOSSA, VALENCIA, &c.

34 maravadies or } =1 real or rial of plate,=10 cents.
8½ quartos } =1 real or rial of velon,=5,31
10 reals of plate=1 Spaniſh dollar,=100 cents.
$15\frac{1}{17}$ reals velon or 8 reals plate=1 peſo, piaſtre or current dollar,=80 cents.
32 reals of velon=17 reals of plate=1 piſtole of exchange= 170 cents.
20 reals velon=1 dollar of plate in Cadiz,=106,25

3 reals of plate=1 pistarine,=20 cents. 1 soldo=6,24 c. 1 libra 125 c. of exchange 110,295 c. 1 pistole of velon and old plate 372 c.

In the provinces of Catalonia livres, sols and deniers are used as the money of account.

25 lb=1 Spaniſh arobe. 1 lb of Spain=$\frac{97}{100}$ lb in U. S. 1 velt=$\frac{33}{16}$ W. gln. 1 fanague=1,52 buſhel. 1 vara=1,08 yds in U. S.

PORTUGAL.

LISBON, OPORTO, COIMBRA, &c.

20 reas or rez=1 vintin or vinten,=2,5 cents.
1000 reas or 50 vintens=1 millrea or milre,=125 cents.

100 reas or 5 vintens=1 testoon,=12,5 cents 10 testoons 1 milrea. 4 testoons=1 crusade of exchange=50 c 24 vintins 1 new crusade, 60 c. 48 testoons 1 moidore, 600 c. 64 testoons 1 joanese 800 c.

32 lb=1 arobe =33 lb in U. S. 4 arobes=1 quintal= 132 lb. 12 canadoes=1 almude,=4½ W. glns. 60 alquieres or 15 fangas=1 moy,=24 buſhels. 1 vara=43⅓ inches, 1 covedo=26⅔ inches, in U. S.

ITALY.

GENOA, ISLE OF CORSICA, Novi, St. Remo, &c.

12 denari=1 ſoldi,=,796 cts. 20 ſoldi=1 pezzo or *15,926 cts.* 5½ lires 1 pezzo of exchange, 92,593 c. 5

1 napoleon, 79,63 cts.=1 croiſade. 6 teſtoons 1 genouine, 137,777 cts. 1 piſtole 318,52 cts.

PIEDMONT, SAVOY, ISLE OF SARDINIA, *Turin*, *Chamberry*, &c.

12 denars=1 ſoldi,=1,389 cts. 12 ſoldi 1 florin, 16,666 cts. 20 ſoldi 1 lire, 27,777 c. 1 ſcudi=100 c. 1 ducatoon 116,666 c. 1 piſtole 361,111 c. 1 louis d'or 444,444.

Milan, Modena, Parma, &c.

12 denari=1 ſoldi,=,313 ct. 20 ſoldi 1 lire, 16,26 cts 115 ſoldi 1 ſcudi current, 92,824 c. 1 philip 97,222 c. 1 piſtole 355,555 c.

TUSCANY, *Leghorn, Florence* &c.

12 denari=1 ſoldi,=,765 cts. 5 quatrini 1 craca, 1,277 c 6 cracas 1 quilo 10,287 c. 20 ſoldi 1 lire 15,43 c. 10 palls 1 tuſcan crown 105,26 c. 1 piaſtre of exchange 92,59 c. 1 ducat 115,74 c. 1 piſtole 344,44 c.

Rome, Anchona, Civita Vecchia, &c.

5 quatrini=1 bayoc,=1,388 cts. 10 bayocs 1 pall,=1 ſtamped julio,=13,888 cts. 10 palls 1 ſcudi of Rome, 133,888c. 1 teſtoon=33,333 c. 1 crown current 111,111 c. 1 crown ſtamped 133,333 c. 1 chequin 2 \$. 1 piſtole $3\frac{4}{9}$ \$. 1 lewe $6\frac{1}{9}$ \$. 1 ſackeen $3\frac{1}{18}$ \$.

Naples, Gaeta, Capita, &c.

3 quatrini=1 grain,=,74 cts. 10 grains 1 carlin, 7,407 c. 10 carlins 1 ducat 74,074 c. 12 carlins 1 Naples dollar, $\frac{8}{9}$ \$. 1 tarin 14,185 c. 1 teſtoon=29,63 c. 1 piſtole 240,741 c.

ISLES OF SICILY AND MALTA, *Palermo, Meſſina,* &c.

20 grains=1 tarin,=8 cts. 12 tarins 1 Sicilian dollar,= 96 cts. 30 tarins 1 onge, 240 c. 1 florin of exchange 48 c. 1 ducat of exchange 104 c.

In *Venice* there are three kinds of money, the banco money is 20 per cent better than banco current, which is 20 pr. ct. better than Picoli money. 12 denari or deniers d'or,= *1 ſoldi or* ſol d'or. $5\frac{1}{6}$ ſoldi=1 groſs or grosi. 24 groſs 1 ducat. 1 ducat banco=92,59 cts. 1 teſtoon $33\frac{1}{3}$ cts. 1 [illegible] *11,11 c.* 1 chequin 203,7 c.

In *Bologna, Ravenna,* &c. 20 bayocs or 2 julios=1 lire,= $22\frac{2}{3}$ cts. 3 julios=1 teſtoon $33\frac{1}{3}$ c. 1 ſcudi of exchange= $94\frac{4}{9}$c. 1 ducatoon $116\frac{2}{3}$ c. 1 piſtole $2\frac{1}{9}$ S.

12 ounces=1 pound in Leghorn, Florence, &c.=$\frac{112}{135}$ lb. avor. 100 lb in Naples=$64\frac{5}{8}$ lb avor. 100 rottoli=1 cantar of Sicily,=176 lb avor. 1 caffis of Sicily=$3\frac{1}{4}$ W. glns. 1 Sicilian barrel=9 glns. 4 braces=1 cane of Leghorn,= $\frac{64}{100}$ yd. 1 palm=$9\frac{1}{2}$ inches. 1 cane of Naples 84 inches.

AUSTRIA AND SUABIA.

Vienna, Trieſte, Augsburg, Blenheim, &c.

4 ſennings or 2 dreyers=1 cruitzer,=,864 c. 60 cruitzers 1 gould, 51,85 c. 2 goulds 1 ſpecie or hard dollar, 103,7 c. 1 rix dollar=77,77 c. 1 ducat 205,55 c. 1 florin of Trieſte 35,55 c. 1 livre of Trieſte=20 ſoldi,=6,66 c.

1 lb. of Vienna=$\frac{123}{100}$ lb. avor. 18 gallons=1 barrel wine. 1 ſtaro of wheet=2,4 buſhels. 1 brace of Trieſte=$\frac{1}{2}$ yd in U. S.

TURKEY.

MOREA, ISLES OF CANDIA AND CYPRUS, *Olympia, Smyrna, &c.*

12 mangars or 3 aſpers=1 parac,=$3\frac{1}{3}$ cts. 10 aſpers or beſtics 1 oſtic, 11,11 c. 20 aſpers 1 ſolota, 22,22 c. 4 ſolotas 1 piaſtre, 88,88 c. 1 caragrough=111,11 c. 1 xeriff= 222,22 c. 40 paras=1 piaſtre in Smyrna=29,41 c.

1 eymar=$22\frac{1}{2}$ lb. 45 okes in Smyrna=100 rottoli=1800 drams=1 quintal $123\frac{3}{4}$ lb. avor. 1 cheque of opium in Symrna=250 drams. 1 tiffee of ſilk=610 drams=41,93 lb. avor. 1 pike=26,9 inches in U. S.

HINDOOSTAN.

Surat, Cambray, BOMBAY, GOA, *Viſapour, Madras, Pondicherry, Calcutta, Callicut, &c.*

400 rees=4 quarters=192 pice=16 annas=1 rupee= 46,5 cts. 80 caſh=1 fanam, 36 fanams=1 pagoda of Madras=165 cts. 4 pagodas 1 gold rupee=660 cts. 1 xeraphim =27,9 cts. 1 tangu=100 c. 1 parco=400 c. 1 fiano= 35,5 c. 100 Sicca rupees=116 current rupees=46,5 S.

Cowries or cori are sea shells used as small money among the natives in India and along the coast of Africa. 4500 or 5000=1 rupee, they are *sometimes sold* by weight, 1 bazar maund for 5, 10 or 14 rupees, varying *according* to the quality.

1 Bombay maud=28 lb. avor. 1 factory maud of Calcutta=75 lb. 1 maud of Madras=25 lb. troy. 1 bazar maud = 84 lb. 1 Surat maud=$37\frac{1}{3}$ lb. 21 Surat mauds=1 Surat candy=784 lb. 100 cattas=1 picul=$133\frac{1}{3}$ lb. 1 cavid= $\frac{1}{2}$ yd.

SIAM, PEGU, MALACCA, CAMBODIA,

ISLES OF SUMATRA, JAVA, &c.

10 cori=1 fette,=,111 cts. 125 fettes 1 ſalteleer 13,88 c. 250 fettes 1 ſooco, 27,77 c. 500 fettes 1 tical 55,55 c. 900 fettes 1 dollar=100 c. 1 rial=$1\frac{1}{9}$ $.=1 ecu=1 crown. 48 ſtivers=1 rix dollar, 75 c. 80 ſtivers=1 ducatoon in Batavia,=125 c.

100 cattas=125 lb. Dutch=1 picul=$133\frac{1}{3}$ lb. avor.

CHINA.

Pekin, Canton, Nankin, &c.

10 caſh=1 candareen=1,25 cts. 10 candareens 1 mace, 12,5 cts. 10 mace 1 tale, 125 cts. 1 rupee=46,5 cts. 1 ecu =111,11 c.=1 crown.

1600 tales=100 cattas=1 picul=$133\frac{1}{3}$ lb. avor. 1 cavid =14,9 inches.

JAPAN.

Jeddo, Meaco, &c.

10 candareens=1 mace,=7,5 cts. 10 mace 1 tale, 75 cts. =1 rix dollar. 6 tales 1 corban, 450 cts. 15 mace 1 ounce of ſilver,=112,5 c. 2 ounces 1 ingot 225 c. 13 ounces of ſilver 1 ounce of gold 14,625 $. 1 japaneſe 29,25 $. 1 double 58,5 $. 1 lattee or cattee=21 oz. of gold,=307,125 $

160 tales=16 mace =1 catta,=1,333 lb. 1 balee=65 wine quarts. 1 ichan or pickey=$3\frac{1}{2}$ ft. in U. S.

PHILLIPPINE ISLES.

12 quartos or 32 maravadies=1 real,=10 cts. 8 reals 1 peſo or dollar,=80 cts. 1 ducat of exchange=110 c. 1 piſtole=355,5 c.

25 lb.=1 arobe,=24,24 lb. avor. $5\frac{1}{2}$ arobes=1 picul.

ISLE OF CEYLON.

6 ſtivers=1 ſhilling flemiſh,=9,3 cts. 8 ſhillings=1 rix dollar 74,4 cts. 30 ſtivers 1 rupee 46,5 c. $64\frac{1}{2}$ ſtivers 1 Spaniſh dollar, 100 c.

PERSIA.

Iſpahan, *Gombroon*, ISLE OF ORMUS, &c.

4 coz=1 biſti,=2,96 cts. 10 coz 1 ſhahee, 7,4 cts. 20 coz 1 mamooda, 14,81 c. 1 larin=18,51 c. 1 abaſhee 29,62 c. 1 or 148,13 c. 1 bovello, 355,55 c. 1 tomond, 1481,48 c.

ARABIA.

Medina, *Mecca*, *Mocha*, *&c.*

5¼ carrets=1 caveer,=1,23 cts. 7 carrets=1 comaſhee, 1,64 cts. 80 carrets 1 larin, 18,74 cts. 18 comaſhees 1 abyſs, 29,52 c. 1 piaſtre=100 c. 1 ſequin 164,01 c. 1 tomond, 15 \$.

EGYPT.

Alexandria, *Cairo*, *Said*, &c.

3 aſpers=1 medin,=30,8 cts. 24 medins 1 Italian ducat 74,07 c. 1 piaſtre=88,88 c. 1 crown=1⅑ \$.=1 ecu. 1 ſultanin 222,22 c. 1 pargo dollar 233,33 c.

BARBARY.

Algiers, *Tunis*, *Tripoli*, &c.

3 aſpers=1 medin,=3,08 cts. 10 aſpers=1 real or rial, 10,2 c. 1 double=25 c. 1 ſilver chequin 74,07 c. 1 zequin 196,29 c. 1 piſtole 372,22 c.

MOROCCO.

Santa Cruz, *Mequenez*, *Tangier*, *Sallee*, &c.

24 fluces=1 blanquil 3,7 cts. 4 blanquils 1 ounce, 14,8 c. 7 blanquils 1 octavo, 25,9 c. 2 octavos 1 quarto, 51,8 c. 1 medio=103,7 c. 1 zequin 200 c. 1 piſtole 372,22 c.

WEST-INDIES, ENGLISH.

JAMAICA, BERMUDAS, BARBADOES, &c.

12 pence=1 ſhilling,=15 cts. in Jamaica and Bermudas, 16 cts. in Barbadoes, 11,11 cts. in Martinico, Tobago and St. Chriſtophers. 20 ſhillings 1 pound. 1 bitt=10 cts. 1 piſtole=372,22 cts. 1 current dollar=91,66 cts. 1 round dollar=100 cts.

WEST INDIES, FRENCH.

ST. DOMINGO, MARTINICO, &c.

12 deniers=1 ſol,=,588 ct. 20 ſols 1 livre, 11,76 cts.

ſols 1 ſcalin, 8,82 c. 1 ecu=108,3 c. 1 piſtole 372,22 c. 1 louis d'or or louis of gold 444,444 c.

Rates at which foreign monies are eſtimated at the Cuſtom-Houſes of the United States.

	Dols.	Cts.
Each pound sterling of Great Britain, at - - - - - - -	4	44
pound sterling of Ireland - - - - - - - - - - - -	4	10
livre tournois of France - - - - - - - - - - -		18¾
florin or gilder of the French Netherlands - - - - -		40
mark banco of Hamburgh - - - - - - - - - - - -		33⅓
rix dollar of Denmark - - - - - - - - - - - - - -	1	
real of plate of Spain - - - - - - - - - - - - -		10
real of vellon of Spain - - - - - - - - - - - - -		5
milree of Portugal - - - - - - - - - - - - - -	1	24
tale of China - - - - - - - - - - - - - - - -	1	48
pagoda of India - - - - - - - - - - - - - - -	1	84
rupee of Bengal - - - - - - - - - - - - - - -		50

Par between the currency of Great-Britain and thoſe of ſeveral other countries, as ſtated by the Encyclopaedia Perthenſis.

PAR, IN STERLING.		S.	D.	PAR, IN STERLING.		S.	D.
Amsterdam	1 rix dollar=	4	6½	Madrid	1 piastre=	3	7
Antwerp	1 ducat	6	3	Malaga	1 piastre	3	7
Augsburg	1 florin	3	1½	Middleburg	1 rix dollar	4	6½
Berlin	1 rix dollar	4	0	Milan	1 ducat	4	7
Bilboa	1 piastre	3	7	Naples	1 ducat	3	4½
Bologna	1 dollar	4	3	Narva	1 rix dollar	4	6
Bolsena	1 rix dollar	3	8	Norway	1 rix dollar	4	6
Bremen	1 rix dollar	3	6	Nuremburg	1 rix dollar	4	6
Breslay	1 rix dollar	3	3	Oporto	1 milree	5	7¼
Brunswick	1 rix dollar	4	6	Osnaburg	1 rix dollar	4	6
Cadiz	1 piastre	3	7	Paris	1 crown	2	5¼
Cologn	1 rix dollar	4	6	Revel	1 rix dollar	4	6
Cracow	1 rix dollar	4	6	Riga	1 rix dollar	4	6
Dantzick	13½ florins	20	0	Rome	1 crown	6	1½
Denmark	1 rix dollar	4	6	Rotterdam	1 rix dollar	4	6½
Dresden	1 rix dollar	4	6	Rowen	1 crown	2	5¼
Embden	1 rix dollar	3	6	Russia	1 ruble	4	5
Florence	1 crown	5	4⅘	Saville	1 piastre	3	7
Francfort	1 florin	3	0	Sicily	1 crown	5	0
Genoa	1 pezzo	4	6	Stettin	1 mark	1	6
Hamburg	1 ducat	9	4½	Stockholm	34⅘ dolls	20	
Leghorn	1 piastre	4	6	Strasburg	1 rix dollar	4	6
Leipsick	1 rix dollar	4	6	Switzerland	1 rix dollar	4	6
Liege	1 rix dollar	4	6	Turkey	1 asper	4	6
Lisbon	1 milree	5	7¼	Venice	1 ducat	[illegible]	[illegible]
Lyons	1 crown	2	5¼	Vienna	1 rix dollar	[illegible]	[illegible]

To aſcertain the true PAR *of Exchange between the currencies of different Countries ; or to try the quality of Metals.*

RULE. Weigh the coin or metal whose comparative value is required, and weigh the same while it is suspended in pure water, from the under side of the balance ; by the difference between those weights, divide the true weight of the coin, and the quotient will be its specific gravity, from which subtract that of the alloy, and the remainder is the proportional quantity of pure (or standard) gold or silver in the given coin ; from the specific gravity of that gold or silver whose value, &c. is known, subtract that of the mixture (or coin whose value is required) and the remainder will be the proportional part of alloy in the coin ; multiply each of these remainders by its specific gravity and the products will be the proportional weight of each quantity of metal in the coin, and as the sum of these is to the real weight of the coin, so is the proportional wt. of each part, to its real wt. then as the real wt. of the coin, whose value is known, is to its value, so is the wt. of the same kind of metal in the given coin, to its real value, in the coin with which it is compared.

From the real value of one piece of coin, that of any other piece or denomination may be easily ascertained, knowing how many or what part of one is equal to the other. The specific gravity of the American standard for gold is about 17,87, and that of silver 10,87 (or so many times heavier than pure water) 24,6268 grains of fine gold=1 D. or 246,268 gr. fine gold and 22,388 gr. alloy=10 D.=1 E. 375,64 gr. fine silver (and 34,14 gr. alloy)=1 D. 27 grains of the standard gold coin of Great Britain and Portugal, 27⅗ gr. of that of France and Spain are estimated at 1 D. in U. S. according to act of Congress. Therefore, 1 gr. English and Portugal gold=3,7037 cts. 1 pwt.=88,8888 cts. 1 oz. troy=1777,7777 cts. 1 lb.=213⅓ D. 1 gr. French and Spanish gold=3,6495 cts. 1 pwt.=87,5884 cts. 1 oz.=17,5176 D. 1 lb.=210,2122 D. 1 gr. fine gold=4,0606 cts. 1 pwt. 97,4546 cts. 1 gr. fine silver=,2662 cts. The alloy usually mixed with silver and gold is copper, about 9 times heavier than water.

What is the true par of exchange between the currencies of Portugal and the U. States ; if the moidore of Portugal contain 170 grains troy wt. of gold, and be equal in size to 10 gr. of water, (or loose 10 gr. or $\frac{1}{17}$ of its wt. while suspended in pure water from the balance, or weigh 100 parts in air and 94,117 in water, the U. S. eagle 158 parts in air and 149,101 in water) The moidore being=48 testoons. 1 joanese 64 testoons. 1 millree=10 testoons. 1 testoon=5 vintens, &c.

Moidore 170 gr.÷10 gr.=17 specific gravity of the moid. 17 -(gravity of alloy) 9=8 qty. of gold in moid. Specific gravity of fine gold 19,637 -17=(qty. of alloy) 2,637. x 9=23,733 wt. of alloy. 8 x 19,637=157,096 wt. of gold. As 157,096+23,733 : 170 real wt :: 157,096 : 147,688 gr. fine gold, x4,0606=599,6+cts.=1 moidore, x 48÷64=1 joanese. 599,6÷48=12,49+cts.=1 testoon, ÷5=2,49 cts.=1 vinten, x 50=1 millrea, &c.

What is the value of 12 pwt. of jeweller's gold, 14 times heavier than water? 14-9=5 qty. of gold, x 19,637=98,185 proportional wt. of gold. 19,237-14=5,637 alloy x 9=50,733. As 50,733+98,185 : 12 pwt :: 98,185 : 7,912-pwt. fine gold, x 97,4546cts.=7,71D.A.

MERCANTILE PRECEDENTS.

PROMISSORY NOTE.

S. 78,45 *Boſton, May 5,* 1810.

For value received, I *A B* promiſe to pay *C D* or order, ſeventy-eight dollars forty-five cents on demand, with intereſt after two months.

Atteſt, E F. A B.

PROMISSORY NOTE BY TWO PERSONS.

500 S 50 *Newburyport, July* 12, 1810.

For value received we jointly and ſeverally promiſe to pay five hundred dollars fifty cents, to *A B* or order, on demand with intereſt. C D.

Atteſt, G H. E F.

" When a note is offered at a bank for discount, two endorsers are generally required, to the first of whom it is said to be payable : Thus —A having occasion for a sum of money, procures B and C as endorsers to his note, and offers it for discount in the following form.

100 S. *Salem, Auguſt* 6, 1810.

For value received, I promiſe to pay *B,* or order, at the Salem Bank, on demand, one hundred dollars, with intereſt after ſixty days. A."

RECEIPT FOR AN ENDORSEMENT ON A NOTE.

Boſton, July 12, 1810. Received from *A B* (by the hand of *C D*) Thirty-eight dollars ſeventy cents, which is endorſed on his note of May 5, 1810.

38 S 70 cts. E. F.

RECEIPT FOR MONEY RECEIVED ON ACCOUNT.

Boſton, January 10, 1810. Received from *A B* (by the hands of *C D*) four hundred and thirty dollars on account.

430 S E. F.

GENERAL RECEIPT.

Salem, March 27, 1810. Received from *A B* ten dollars twenty-nine cents in full of all demands.

10 S 29 cts. C. D.

"A general receipt will discharge all debts, except such as are on specialty, that is, bonds, bills and other instruments that may properly be called acts or deeds, viz. those that require to be executed in a solemn manner, where the sealing and delivery are the moſt essential parts of the act, and on that account can only be destroyed or cancelled by something of equal force, viz. some other specialty, such as a general release, &c. Neither will it discharge endorseable promissory notes, or inland bills."

BILL OF EXCHANGE.

Newburyport, Feb. 12, 1810.

EXCHANGE for £ 1000 ſterling.

Sixty days after ſight (or at uſance) pay this my firſt of exchange (ſecond and third of the ſame tenor and date not paid) to A B or order, One Thouſand Pounds Sterling, with or without further advice from

Your humble ſervant, C. D.

Meſſrs. E & F, Merchants, London.

BILL OF GOODS.

Mr. A B bought of C D, *Boſton, May* 5, 1810.

		$.	Cts.
32 ells of mode . . . at . . .	40 cts.	12	80
64 yds. ſtriped Nankins	30	19	20
28 — ſtriped Callico	52	14	56
4 pieces Ruſſel	375	15	00

Received his note payable in 60 days, C D. $ 61 56

ACCOUNT RENDERED.

Mr. A B to C D Dr.

1810.				$.	Cts.
Jan.	5	For 1 barrel of flour		10	
	8.	4 lb coffee at	33 cts.	1	32
	9.	9 lb of ſugar	15	1	35
	23.	7 gallons of molaſſes	42	2	94
Feb.	7.	3 quintals of fiſh	250	7	50
	16.	2 lb hyſon tea	141	2	82
Mar.	29.	5 lb chocolate	25	1	25
May	5.	2 buſhels of corn	80	1	60
				$28	78

Salem, June 6, 1810.

Rec'd payment for C D. E. F.

Dr. Mr. A B IN INTEREST

1810	$			days.	
To Int. on	35	from Jan. 31,	to Oct. 12,	256	$ 1,47
do. on	2962,19	Feb. 2,	to do.	254	123,68
do. on	2590,42	May 31,	to do.	134	57,06
do. on	1733,97	July 2,	to do.	102	29,07
do. on	73,63	July 12,	to do.	92	1,11
do. on	455,52	Aug. 25,	to do.	47	3,51
do. on	158,71	Sept. 30,	to do.	12	0,31
					$ 216,21

Dr. *Mr. A B in account current with C D.* Cr.

1810.		$ cts.	1810.		$ cts.
May 19.	To 1 barrel flour delivered E	9 50	Aug. 10.	By amount of repairs on ship America	57 75
28.	- Cash delivered F	28 50		By Cash	20
June 20.	- 5 lb butter 87c. tea 100 E	1 87			
27.	- 3 quintals fish F	7 50			
July 2.	- ½ barrel Flour F	5			
15.	- Order paid to G	25 50			
		77 87			77 87

Boston, Aug. 10, 1810. Errors excepted, C D.
12 cts.

INVOICE.

INVOICE of merchandize on board the brig Swan, A B maſter, ſhipped by C D on his own account and riſk, for the Weſt Indies, and conſigned to ſaid maſter for ſales and returns, viz.

	140 M. of boards and plank,	10 $	1400 $
	20 M. of white-oak hhd ſtaves	30	600
	12 M. of red-oak hhd do.	12	144
	130 M. of ſhingles	3	390
B.No.1 to 18.	18 hhds of cod-fiſh, 17303 lb	4 pr. C.	692 12
B.No.1—52.	52 bbls of beef	12	624
E No.1—30.	30 bbls of ſalmon	10	300
F.No.1—7.	7 caſks of rice, neat 39,94 C.	4pr cwt	159 76
G.No. 1 & 2.	2 bbls. pork	18	36
	3 M of hoops	25	75
	1300 pair of ſhoes	50 cts	650
			$ 5070 85

Portſmouth, Sept. 7, 1810.
Errors excepted. C D.

ACCOUNT OF SALES.

SALES of 20 *hhds.* 7 *bbls. and* 31 *bags coffee, for and on riſk of Mr. A B, merchant in Portland.*

1810.			
March 15.	C D 20 hhds. wt. 14376 lb. at 23 cts. per lb.		$ 3306 48
16.	E F, 7 bbls. wt. 1493 at 23 cts.		343 39
17.	G H, 31 bags, 5507 23		1266 61
			4916 48
Charges,			
	Advertiſing,	$ 1 46	
	Storage,	3 50	127 87
	Commiſſion on 4916 $ 48 cts at 2½ per cent.	122 91	
	Neat proceeds paſſed to his credit.		$ 4788 61

Errors excepted, &c.

BILL OF SALE.

"TO all people to whom this present Bill of Sale shall come, I, A B of Newburyport, in the State of Massachusetts, Merchant, send Greeting: KNOW YE, That I the said A B, for and in consideration of the sum of THREE THOUSAND, TWO HUNDRED AND TWENTY-TWO DOLLARS, to me in hand well and truly paid at or before the ensealing and delivery of these presents, by C D, of the said Newburyport, Merchant, the receipt whereof I do hereby acknowledge and am therewith fully and entirely satisfied and contented, have granted, bargained, and sold, and by these presents do grant, bargain and sell, unto the said C D, all the hull or body of the good brig Sally, together with all and singular her masts, spars, sails, rigging, cables, anchors, boats and appurtenances, now lying at Newburyport, and registered at the port of Newburyport, the certificate of whose registry is as follows:

IN pursuance of an Act of the Congress of the United States of America, entitled, "An ACT concerning the registering and recording of ships or vessels," A B of Newburyport, in the State of Massachusetts, Merchant, having taken ot subscribed the oath required by the said act, and having sworn that he is the only owner of the ship or vessel called the Sally, of Newburyport, whereof E F is at present master, and is a citizen of the United States, as he hath sworn, and that the said ship or vessel was built at Salisbury, in the said state, in the year seventeen hundred and ninety-nine, as also appears by a certificate of enrolment, No. 129, issued in this district on the fourth day of *August last, now surrendered*—and G H Surveyor of this district, hav*ing certified* that the said ship or vessel has one deck and two mas and that her length is sixty-nine feet five inches, her breadth twenty-two feet and one half inch, her depth eight feet two inches, and that she measures one hundred and sixty tuns and forty ninety-fifths, that she is a square-sterned brig, has no galleries and no figure head, and the said A B having agreed to the description and admeasurement above specified, and sufficient security having been given according to the said act, the said brig has been duly registered at the port of Newburyport.

Given under my hand and seal at the port of Newburyport, this first day of January, in the year one thousand eight hundred and ten.

To have and to hold the said granted and bargained brig Sally and premises with the appurtenances, unto the said C D his heirs, executors, administrators or assigns, to his only proper use, benefit and behoof forever. And I the said A B do avouch myself to be the true and lawful owner of the said brig and appurtenances, and have in myself full power, good right and lawful authority to dispose of the said brig as aforesaid, and her appurtenances in manner as aforesaid, and furthermore, I the said A B do hereby covenant and agree to warrant and defend the said brig and premises, with the appurtenances against the lawful claims and demands of all persons whatsoever unto the said C D. In witness whereof, I the said A B have hereunto set my hand and seal, this tenth day of June, in the year of our Lord one thousand eight hundred and ten.

BILL OF LADING.

A. B.
1 a 53
Casks Pot Ash.
tun. cwt.
8 18 l. s. d.
at 80 s.—35 12 0
Primage 5
pr. ct 1 15 7
———
£. 37 7 7
———

"SHIPPED in good order and well conditioned by A B, in and upon the good ship called the Iris, whereof is master for this present voyage C D, and now riding at anchor in the harbor of Newport, and bound for Liverpool, to say, fifty-three casks of pot ash, containing eight tuns and eighteen cwt. being marked and numbered as in the margin, and are to be delivered in the like good order and well conditioned, at the aforesaid port of Liverpool (the danger of the seas excepted) unto Mr. E F or to his assigns, he or they paying freight for the said goods, four pounds British sterling per ton, with five per cent. primage. In witness whereof, the master or purser of the said ship hath affirmed to three bills of lading all of this tenor and date, the one of which being accomplished, the other two to stand void. Dated at Newport, July 7th, 1810.

C D.